INCOMPLETELY SPECIFIED COMBINATORIAL AUCTION: AN ALTERNATIVE ALLOCATION MECHANISM FOR BUSINESS-TO-BUSINESS NEGOTIATIONS

By

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The popularity of auctions has increased dramatically with their introduction to the Internet. The migration has provided a unique opportunity to harness the power of computing to create new auction forms that were previously unworkable. This research presents a new auction mechanism designed to accommodate the potentially large and often complex problems that are commonly reflected in negotiated sales. I describe a new and innovative way of using an auction mechanism by modifying a combinatorial auction to accept inexact multi-criteria package bids. The bids are incompletely specified yet provide enough of a framework to guide, rather than dictate, the choice of goods to satisfy stated needs. The ability of the bidder to prescribe various aspects of the sale, beyond her willingness to purchase goods at a particular price, makes it possible to use this type mechanism to replace or enhance negotiated sales.

The allocation of goods requires solving a complex combinatorial problem in real-time. In the past this has been considered completely impractical in a conventional auction setting. Utilizing computing resources an online auction of this nature is not only feasible but may provide a way to optimally allocate a given set of bids while satisfying bidder preferences. This auction is applicable to any collection of goods but is most appropriate for complementary goods. As expected, the proposed model becomes computationally intractable as the number of bidders increase, I therefore present simplifying heuristics to make large problems manageable.

Television advertising sales provides an interesting arena in which to investigate the use of the incompletely specified auction. The auction will be employed to accommodate sales where approximately 300 to 350 buyers compete for a finite amount of commercial airtime in the upcoming season. Several constraints unique to media buying are included in the model increasing its complexity significantly which include separation of competing ads, meeting bidder's demographic group exposure requirements while ensuring the seller receives her reservation prices, allowing for a variety of commercial lengths, and accommodating specific show placement requests.

CHAPTER 1
INTRODUCTION

1.1 Introduction

Electronic or Internet based auctions have garnered a great deal of interest in recent years. The renewed popularity of auctions stems from various characteristics unique to this form of commerce. Web based auctions enjoy a much broader audience due to their accessibility to anyone with an Internet connection, thereby growing the buyer pool, increasing competition and thus enhancing profits. The expense and logistics of gathering at one location, a major deterrent to conventional auctions, has been replaced with inexpensive websites. Barriers to entering the electronic auction market have been lowered for all participants. Finally, electronic auctions provide us an opportunity, through harnessing the power of computing, to establish more complex trading rules and handle more complex goods. It is this opportunity upon which we attempt to capitalize in this research.

Recent progress in utilizing computing and networking power includes a variety of new auction designs to facilitate the simultaneous sale of multiple items. Various mechanisms have emerged such as the popular Internet based Yankee Auction (Vakrat & Seidman, 1998) and the Groves/Vickery design (Bapna, Goes & Goupta, 1998) in which a specified number of identical items are offered for sale simultaneously with the items

going to the top bidders whose aggregate demand equals the number of items for sale. Alternatively, the FCC Spectrum license sales of the early 1990s showed that by executing single items auctions simultaneously, buyers could aggregate a desired collection of goods (Crampton, 1995; McMillan, 1994).

None of the above designs allow the bidder to submit a single bid for a combination of heterogeneous items, although it has been shown that in the non-auction environment the value of a bundle of positively correlated goods can be greater than the sum of the individual item values (Bakos & Brynjolfsson, 1998). Allowing the bidder to create a unique bundle of desired goods for which a single bid is submitted, referred to as a package bid, would seem like a logical way to improve the efficiency of auctioning complementary goods by capturing the synergies between product offerings. The most recent advance in auction design, the combinatorial auction, acknowledges this need and profits from the use of computing power. In the past bidding on a group of objects has been considered completely impractical in a conventional setting due to the complexity of winner determination, however utilizing computing resources an online auction of this nature is not only feasible but has proven superior to other multi-object sales mechanisms (DeMartini, Kwasnica, Ledyard & Porter, 1999).

All of the auction forms mentioned so far suffer from the fact that the buyer supplies a bid restricted to item and price information. For example, most combinatorial auctions assume the bidders desire multiple goods and have a reservation price for each, thus their package bid consists of the desired quantity of each item and a per unit price, or a single price for a designated collection of items. In either case a price-item vector is the sole basis for allocating winning bids. Constraints limiting the allocation, within the

current combinatorial formats, are generally restricted to meeting reservation prices and product availability. In many cases this does not adequately reflect the needs of either the buyer or seller. Negotiated sales is a prime example of a business process that could benefit from the use of auctions as they provide an effective means of price discovery, especially for products hard to price a priori or when information asymmetries are present (Englbrecht-Wiggans, 1980; Milgrom, 1989; Choi & Whinston, 1998). However, incorporating the negotiation process into an auction mechanism requires the bid to contain extended specifications. The complexities generated by the required modifications to existing auctions have discouraged widespread use of electronic negotiation models (Choi, Stahl & Whinston, 1997). This research attempts to develop a new auction mechanism that captures the intricacies necessary to enhance or replace a negotiated environment.

In this chapter we provide an overview of our thesis, including a brief description of the mechanism and application environment, our motivation and the research contributions. The research problem is presented in Section 1.2, the motivation for developing the particular model is discussed in Section 1.3 and the anticipated impact of this study is provided in Section 1.4.

1.2 The Research Problem

This research attempts to develop an auction mechanism as an alternative to sales usually accomplished through negotiation. The mechanism will need to provide the buyers with an opportunity to constrain the allocation of goods through a multi-criteria package bid. To impart flexibility, the mechanism must allow bids that are incompletely

specified yet provide enough of a framework to guide the allocation. The ability of the bidder to dictate various aspects of the sale, beyond her willingness to purchase units at a particular price, makes it possible to use this type of mechanism to replace or enhance a negotiated environment.

Our research problem is to design an auction mechanism to accommodate the potentially large and often complex problems that are commonly reflected in the negotiated environment. We address this problem by modifying a combinatorial auction to accept inexact multi-criteria bids. The bids must extend beyond the current price-item vector and allow the bidder to guide, rather than dictate, the choice of goods to satisfy stated needs. We address this problem by carefully constructing a mechanism to accommodate the requirements of an industry whose sales are currently conducted exclusively through negotiation.

The selection of winning bids in a combinatorial auction is an extremely complex problem, in fact it has been shown to be NP-complete (Rothkopf, Pekec and Harstad, 1998). Therefore, another question addressed in this study is to ascertain if a heuristic allocation engine can be developed to determine a satisficing solution in real time and if so how effective is it?

1.3 Motivation

The primary motivation for studying this problem was an acknowledgement of the need for an auction mechanism that more accurately reflects the demands of the market. Rarely are purchase decisions predicated solely on the price of an item, yet current auction mechanisms make allocation decisions based on this limited criterion. Secondly,

the ability to harness computing power has made possible more complex auction mechanisms opening an exciting area of research and we want to expand the models available through this endeavor.

Finally, we have identified an industry that may benefit from changing their current business process. We design our auction mechanism to sell commercial airtime for the Network Television Industry. Television advertising airtime is a commodity product that is currently sold through negotiations that are "frequently based on long-term relationships and editorial and demographic synergies, not just getting the lowest price (Weaver, pp.1, 1999)." Units are typically allocated on a first come first serve basis as opposed to being dictated by competitive forces that could enhance the network's ability to achieve an equilibrium based distribution of goods. The complexity of determining an allocation that simultaneously satisfies the market participants' demands restricts the seller's ability to promote competitive bargaining. This complexity is evidenced by advertising agencies widespread use of "optimizer," decision support software to assist in planning and buying media. Optimizers are purported to give knowledge and real-time information to the buyers to help them determine the best mix of media, i.e. network (including various daypart decisions), cable, syndication, billboard and print (Ross, 1998). Some suggest that if networks do not embrace technology they will get left behind (Stewart, 2000). The major networks have already seen an erosion of their market share with cable and syndication among the beneficiaries (Ross, 1998).

Further evidence of the need for change in this industry is the appearance of alternative selling mechanisms. A number of web-based auctions have appeared selling excess last minute advertising inventory. These sites, such as AdOutlet.com and

Adauction.com, are simplistic in nature selling single units of time that are considered "fire sale" spots or unsold airtime within close proximity to airdate. Airtime is similar to airline seats, at the end of each day unsold commercial airtime can never be recovered. These sites have been criticized for their limited offerings and their focus on "distressed" inventory (Stewart, 2000; Coleman, 1999). Proponents point out the advantage of 24-hour access and price discovery afforded by the sites (Kuchinskas, 1999).

Current online advertising auctions are designed for scatter and opportunistic sales that handle short-term campaigns and or supplemental purchases throughout the year. There is no mechanism designed to assist in "upfront sales," the large onetime sale of spots encompassing annual campaigns. This market provides an interesting arena in which to extend the current design of the combinatorial auction. The combinatorial auction is appropriate for this environment to accommodate synergies between products and consumers' desires for a collection or bundle of goods to meet their annual campaign exposure requirements. In this environment there are also ample substitutes so buyers are not restricted to obtaining a specific item but may be satisfied with any number of the substitutes available. The current versions of combinatorial auctions do not allow for substitutes or bids that do not precisely specify the desired objects. Therefore, a new mechanism needs to be developed to accommodate these market characteristics. Chapter 3 explores this industry in greater detail, including a discussion of constraints imposed on the placement of ads that must be conveyed through the mechanism.

1.4 Expected Contributions of this Research

Two major contributions are expected from this research. First, we hope to develop a new business-to-business auction mechanism, the generalized model will be germane to environments were the purchase decision criteria extends beyond the price of the item or items desired. This multi-dimension combinatorial auction will accommodate business models that currently use negotiations as their primary sales mechanism and sales for products and or services that require multi-criteria decisions. The other major result from this study is the development of a heuristic designed to determine auction winners and efficiently allocate inventory. We hope the heuristic will provide an optimal or near optimal allocation and thus provide a methodology applicable to other combinatorial optimization problems.

1.5 Summary

This chapter has briefly introduced the research project and discussed the merits of the study as well as its anticipated contribution. The remainder of this dissertation is organized as follows. Chapter 2 will present the background literature for auctions, including an overview of classical theory as well as recent discoveries propagated by the changes wrought by the migration of auctions to the Internet. Chapter 3 will introduce the application environment and define the problem using an integer programming model. We review constraint programming, a methodology effective in solving the type of problem represented by our auction in Chapter 4. The fifth chapter describes the heuristic development in detail. We intend to test our mechanism's performance through

simulation and in Chapter 6 we present the characteristics of the artificial agents that will represent bidders in our experiments. The agents are modeled to be representative of the industry participants, their characteristics reflect patterns discovered from data provided by a major corporation in the television industry. In Chapter 7 we outline the experiments that will be conducted to judge the performance of the mechanism. Details of experimental results as well as an interpretation of the findings are given in Chapter 8. Finally, conclusions from the research and identification of future directions for this study encompass the remainder if this dissertation.

CHAPTER 2
AUCTION THEORY

2.1 Introduction

This chapter presents a general overview of Auctions. The goal of this chapter is to review classic auction theory and various extensions that have originated from it. Section 2.2 will provide a general overview of auction theory, specifically defining an auction, its players, various auction categories and the settings that favor their use. Section 2.3 describes various auction types, both single item and multiple object formats. Section 2.4 will present the well accepted classical or "benchmark" auction model. Section 2.5 looks at modeling issues that must be considered and broadens our perspective by reviewing several auction situations that either extend or restrict the benchmark model. Section 2.6 reviews the impact of the Internet on auctions.

2.2 Overview

"An auction is a market institution with an explicit set of rules determining resource allocation and prices on the basis of bids from the market participants" (McAfee & McMillan, 1987, pp.701). It attempts to match a buyer with a seller to achieve a market clearing equilibrium price (Engelbrecht-Wiggans, 1980). Formalized trading procedures govern the players' interaction based on specific rules for competitive bidding and trade execution (Klein & O'Keefe, 1998). Choi and Whinston (1998) describe an

auction as simply posted prices where the price movements are more rapid and the number of participants greater.

Auctions are the simplest and most familiar means of price discovery (Engelbrecht-Wiggans, 1980). Thus, they are effective when items are hard to price (Beam & Segev, 1998), for instance when demand for an item is difficult to determine *a priori* or when a product's value varies greatly (as with time sensitive products or services, which tend to lose their value rapidly) (Chio & Whinston, 1998; Bapna et al.,1998; Milgrom, 1989). Auctions are commonly classified by the goods traded or the rules used that determine the final price (Lengwiler, 1999). Some auctions are performed in real time while others accept bids over time to be matched at a specified later date (Klein & O'Keefe, 1998).

A typical auction consists of four major components: players, objects, payoff functions and strategies (Engelbrecht-Wiggins, 1980). Players include the bidders, the seller and the auctioneer. Most literature views auctions from the perspective of the seller who owns the item(s) for sale and is attempting to maximize his profit. Conversely, the bidder attempts to minimize the price, thereby maximizing her utility. The seller, as the Stackelberg leader or first mover, normally precommits to a set of policies, choosing the auction form and rules (McAfee and McMillan, 1987). A Stackelberg leader is implicitly assumed to commit first to his chosen action and not change that action after receiving additional information, even though changing might be profitable *ex post* (Rasmusen, 1989). This seeming advantage is tempered by information asymmetries. The seller and buyers do not know the other player's valuation of the object to be auctioned (Choi &

Whinston, 1998). The auctioneer, in the traditional setting, is simply a facilitator bringing together the buyer and seller.

The number of items offered, value information, physical characteristics and type characterize objects (Engelbrecht-Wiggins, 1980). Single objects are the common focus of classical auction theory, however objects may be single divisible or indivisible items, a package of non-identical items, or multiple homogeneous items. Value information broadly defines who knows what. There are two commonly assumed models, the Independent Private Value model (IPV) and the Common Value model. If bidders know with certainty the value they individually place on an item the auction uses the Independent Private Values model. Individual valuations have a common distribution but are statistically independent of the other customers' valuations (Beam, Segev & Shanthikumar, 1996). A Common Value object, on the other hand, is assumed to have a single value but information regarding this value is varied among bidders. The bidders' valuation is dependent on at least one common objective variable, possibly resale value or future royalties (Kagel, 1995; Das & Sundram, 1997; Milgrom, 1989). Due to the statistical dependence inherent in the Common Value model, bidders tend to infer information from others' bids (Beam, et al., 1996). Milgrom and Weber (1982) refer to the positive correlation between bidder valuations in the Common Value model as affiliation.

The payoff function involves decisions surrounding the financial transfer of a product such as the award mechanism or the rule used to determine the winning bid, final price and recipient, the presence or absence of a reservation price, and other participation costs (Engelbrecht-Wiggins, 1980). Occasionally included in the calculation of the

payoff function are charges for preparing and submitting bids and participation fees. The price paid by the winner for the object can depend solely on the final bids or on something correlated with the item's values such as royalties (McAfee & McMillan, 1987). In single object auctions, the payoff function routinely awards the object to the highest bidder if that bid exceeds the seller's reservation price. The seller has the right to retain the item should bidding fall below a predetermined minimum amount, referred to as his reservation price (Milgrom, 1989).

Finally, strategies refer to how a bidder executes her bid. Milgrom (1989) defines a "pure" bidding strategy as one that is based on a function of the information the bidder knows. If all bidders accurately predict the other participants' bidding strategies and then use that information to select their own strategy, Nash Equilibrium is achieved (Nash, 1950). Nash Equilibrium (or rational-expectations) strategies maximize the expected utility of the outcome and is used routinely as the strategy of choice in classical auction theory (Engelbrecht-Wiggins, 1980).

2.3 Auction Types

2.3.1 Single Item Auctions

There are four basic types of auctions used when a unique item is bought and sold. They can be classified by the rules that govern the exchange of goods. The rules affect bidding strategies and incentives and thus transaction efficiency.

English Auction: By far the most common type of auction, the English auction is an oral, outcry, ascending auction where progressively higher bids are solicited until only one bidder remains. The object is awarded to the remaining bidder at the price equal to

her highest bid. At equilibrium the bidder who values the item the most will retain the object at a price equal to their second highest valuation, therefore the English auction is efficient (Milgrom, 1989). Due to the open nature of the auction, bidders can observe the behavior of other bidders, process this information and dynamically modify their reservation prices (under the Common Value model) (Beam, Segev and Shanthikumar, 1996). The dominant bidding strategy used is to bid until the price exceeds the buyer's willingness to pay, normally a small increment below the bidder's true valuation (Beam, et al, 1996). Examples include art, antique and livestock auctions.

Dutch Auction: Similar to the English auction, the Dutch auction is an open outcry, oral auction. It differs only in the direction of the bid progression. In a Dutch auction the auctioneer calls out an initial high price and then successively lowers the price until a bidder claims the object, normally by shouting "mine!"(Choi & Whinston, 1998). Unlike the English auction, since the auction concludes with the first bid, bidders cannot gain signal information from the behavior of other bidders (Beam et al., 1996). The dominant bidding strategy used is to claim the object when the bid equals a small increment below the bidder's true valuation (Beam et al., 1996). The increment between the bidder's true valuation and the actual bid is the buyer's surplus. The cut flower market is an example of this type of auction.

First Price Auction: This mechanism is also known as the First Price Sealed Bid Auction. Potential buyers submit a single sealed written bid. Bids are opened simultaneously and the item is awarded to the buyer who submitted the highest bid at a price equal to her bid (Milgrom, 1989). As in Dutch auctions, information cannot be gleaned from observing other bidders' behavior, therefore bidding strategies are based on

the participant's individual value of the object and the expected behavior of the other customers (Beam, et al., 1996). This auction may be inefficient since the bidding strategy employed is affected by information asymmetries and thus may result in not awarding the item to the highest value bidder (Milgrom, 1989). Government procurement contracts commonly use this auction type.

Second Price Auction: This is also known as the Vickrey or sealed-bid second price auction. As with the first price sealed bid auction, buyers submit sealed bids with the highest bidder claiming the object. However, as the name implies, the price paid for the item is equal to the second highest bid rather than the winning bid (Vickrey, 1961). Bidding one's true valuation is the dominant strategy for this auction since the object will be awarded at some increment below the winning bid, ensuring consumer surplus (Engelbrecht-Wiggans, 1980). The Vickrey auction duplicates the principle characteristics of the English auction and, due to the similar theoretical properties discussed in depth in section 2.4, as well as notational efficiency, English auctions are customarily modeled as Second Price auctions (Milgrom, 1989).

Variations of the four general categories have spawned a multitude of auctions. For example, fees can be charged for participation or the seller may impose a reservation price below which he will not sell (McAfee & McMillan, 1987).

2.3.2 Multiple Item Auctions

Vickrey (1961) first proposed the "multiple auction" where several identical units are sold to bidders who desire but one item. Two variations of the progressive auction, simultaneous and sequential, are most commonly used to facilitate multi-unit sales. How

the auction(s) are conducted within the multi-unit environment (i.e., sequentially or simultaneously) propagate a variety of alternative designs. Additionally, recent progress has been made in facilitating package bidding within a multi-object variation of the traditional English auction.

Sequential auctions utilize one of the standard or modified auction forms but execute them serially until all goods have been exhausted (Vickery, 1961). Problems exist with sequential auctions making them less attractive than simultaneous auctions. First, they reduce the bidder's ability to efficiently aggregate. Once an auction is complete the items are allocated, if in subsequent auctions the bidder is unable to obtain a complementary product, the value of the previously acquired object diminishes. With the sequential auction you can not modify earlier bids. McAfee and McMillan, (1996) and Ashennfelter (1989) also discovered that sequential auctions produce various prices for identical items depending on their position in the sales sequence (later sales tend to fetch greater prices). Finally, bidders that are budget-constrained can be eliminated by one or a group of bidders driving up the price in the early rounds, thus effectively exhausting the constrained budget (Benoît & Krishna, 1998). Revenue comparisons conducted by Benoît and Krishna (1998) show that sequential auctions have appropriate applications. Sequential auctions were discovered to outperform simultaneous if objects are substantially different or if the complementarities are significant (Benoît & Krishna, 1998; Krishna & Rosenthal, 1995).

Alternatively, a collection of auctions equal to the number of objects could be held simultaneously. Simultaneous ascending auctions are covered in detail by McMillan (1994), Krishna and Rosenthal (1995), Crampton (1995), McAfee and MccMillan (1996),

and Milgrom (1997). With simultaneous ascending auctions bidding is open for multiple items at the same time and remains open as long as there is active bidding on some unit. Bidding occurs in rounds with the results posted at the end of each (Milgrom, 1997). Information access allows participants to analyze their position with respect to the bundle of items they hope to acquire. Keeping all auctions open gives a bidder the flexibility to aggregate products in the manner she chooses and reconfigure those choices should the combination become too expensive (McAfee and McMillan, 1996). Arguably the most widely known form of multi-object simultaneous auction are the FCC Spectrum auctions. The government allocated PCS narrowband licenses using a simultaneous ascending auction, dubbed “the greatest auction in history” by the *New York Times* which was designed specifically to facilitate the aggregation of multiple licenses by a single buyer (Benoît & Krishna, 1998). The positive synergy between licenses in adjoining areas was acknowledged by the architects, but combination or package bidding was not allowed due to its perceived complexities (Cramton, 1995). Simultaneous auctions are subject to an exposure problem, a phenomenon causing bidder losses (DeMartini, et al. 1999). Losses result from "mutually destructive bidding" where bidders unable to obtain a complete set of goods due to competition are left holding goods priced at more than their value (Bykowsky, Cull & Ledyard, 1995). To protect themselves, bidders may bid less aggressively precipitating reductions in efficiency. To overcome this problem investigators have suggested allowing package bids (Ausubel, Cramton, McAfee & McMillan, 1997).

Very little work had been done involving bundling of auction goods until the FCC auction design debate highlighted an assignment void involving auctions for multiple

heterogeneous goods with synergies. Palfrey (1983) had analyzed bundling decisions for multiple heterogeneous objects where demand was uncertain and found that seller surplus diminishes as the number of bidders increase. Information asymmetries led the seller to bundle items for which higher individual prices could have been obtained. Kim (1996) later derived an equilibrium model in which bidders incorporate their individual and complementary valuations so as to ensure that the auction is efficient with respect to the bundled value while not actually auctioning the bundle as a whole. Enlisting the help of electronic agents, Fan, Stallaert and Whinston (1998) introduced “bundle trading” to effectively construct investment portfolios.

If complementarities exist among the items being sold, evidence suggests that it may be more appropriate to permit bidders to bid for packages, rather than simply bidding item by item such as in the FCC auction (Banks, Ledyard & Porter, 1989; Bykowsky, Cull & Ledyard, 1995; McMillan, Rothschild & Wilson, 1997; Ledyard, Porter & Rangel, 1997; DeMartini, Kwasnica, Ledyard & Porter; 1999). Unlike the work of Palfrey (1983) where the seller creates the bundle of goods, bidders form the bundles by submitting package bids in what is termed a "combinatorial auction." One of the first investigations into this auction format was conducted by Rassenti, Smith and Bulfin (1982) and allowed package bidding to allocate airport time slots to competing airlines. They employed a "set-packing" algorithm to determine resource shadow prices later used to ensure package prices fell at or below bid amounts. Empirical tests showed the mechanism to be efficient and demand revealing. Rothkopf, Pekeč and Harstad (1998) describe this type of auction as one that facilitates a bidder's desire to submit bids for combinations of assets.

Three newly designed implementations of the combinatorial auction, the Adaptive User Selection Mechanism (AUSM), the Resource Allocation Design (RAD) auction and a Web-base implementation are being investigated to meet multiple item allocation needs when synergies between or among objects exist. The Adaptive User Selection Mechanism is a modification of the English ascending-bid auction that allows both package bids and individual item bids. Described by Banks et al. (1989) as a decentralized mechanism, with continuous bidding communicated via an electronic bulletin board. Initial implementations suffered from a new phenomenon, the threshold problem. It was discovered that when package bids are allowed, small bidders may not be able to dislodge a large but inefficient package bidder (see DeMartini et al.(1999) for an excellent description). A "standby" queue was added to facilitate coordination among bidders to form a large enough collection of small bids to displace the current winner(s) (Banks et al., 1989). Although the queue solved the threshold problem, it added complexity to the mechanism. The Resource Allocation Design (RAD) Auction simplifies the AUSM by introducing a new pricing rule eliminating the need for the standby queue and thereby overcoming the AUSM's complexity issue. A vector of single item prices is internally computed from bids and used to check minimum bid increment requirements and convey information to bidders. Bid opportunities are presented to bidders combating the formation of thresholds (DeMartini et al., 1999). Finally, Teich Wallenius, Wallenius, & Zaitsev (1999) offer an electronic auction for multiple homogeneous units that allows the bidders to specify if they will accept partial fulfillment of their package bid. Additionally, due to its semi-closed nature, sellers can establish various reservation prices for different quantity levels thus facilitating price

discrimination. The auction is sealed in that bidders do not have access to the bids of others but the mechanism recommends entering bids to inactive bidders, or bids that based on current conditions would be a potentially winning bid. This design has been shown to overcome the "winners' curse" prevalent in common value auctions.

Other problems faced in this arena stem from the sheer number of possible bundle combinations. In light of the complexity, Rothkopf et al. (1998) investigate how to determine a revenue maximizing set of non-conflicting bids and identify structures that are computationally manageable. Placing certain restrictions on the family of permitted bids formed the basis of their analysis. Nested combinatorial bids that form a single tree structure provide, through "rolling back" the tree, a straightforward way to determine the revenue maximizing outcome (Rothkopf et al., 1998). Additionally, if bid combinations are composed of either at most doubletons or at least a large proportion of the number of available assets, maximizing algorithms could be found that are mathematically tractable. Bids with the intervening cardinalities are considered NP-complete (Rothkopf et al., 1998). Some geographic structures such as an interval of consecutive assets or items that could be organized into a k-dimensional matrix, thereby permitting row or column-wise bids, were proven by Rothkopf et al. (1998) to be computationally manageable.

2.4 Framework: The Benchmark Model

The next logical question to be answered is which of the auction forms is optimal? Bulow and Roberts (1989, pp. 1060) define an optimal auction as a "bidding mechanism designed to maximize a seller's expected profit." This complex topic has received a great deal of scholarly attention including work by Vickrey (1961), Myerson (1981), Bulow &

Roberts (1989), Riley and Samuelson (1981), Milgrom and Weber (1982). We will begin our analysis by presenting a simple framework.

The following standard assumptions, gathered by McAfee and McMillan (1987) from early game theory models such as those defined in Vickrey's 1961 seminal paper, define the "Benchmark Model." The assumptions are common knowledge among the participants.

1. All participants are risk neutral (bidders and sellers).
2. Bidder valuations are independent and private (Independent Private Value Model). The value (v_i) that bidder *i* places on an object is independently drawn from a distribution F_i. Note: although the individual values are private, all players know the distribution governing their valuation.
3. Bidders are symmetric, every buyer has the same cumulative distribution denoted by F.
4. There are no fees associated with the auction. The price paid by the winning bidder is dependent entirely on the bids themselves.

Based on these fairly restrictive assumptions, each of the four basic auction types results in the same expected revenue. This notion is the basis of the Revenue Equivalence Theorem which states that for the benchmark model, the four standard auction forms yield the same price on average (Das & Sundram,1997; Milgrom, 1989; Vickrey, 1961; Ortega-Reichert, 1968; Myerson, 1981; McAfee & McMillan, 1987). Fueling the field of optimality study are modifications to the basic or benchmark assumptions.

2.5 Modifying The Benchmark Model Assumptions

There are several modeling issues that warrant consideration when choosing particular auction rules. Most, but not all, correspond to the benchmark model assumptions of risk aversion, value formulation, bidder asymmetry, fees and the number of items to be sold. Strength of bidding competition and the potential for collusion also must be addressed.

Uncertainty is central to choosing to utilize an auction as the sales mechanism. Should the seller possess perfect information, the need for price discovery would be moot and posted prices would optimize the seller's surplus (McAfee & McMillan, 1987). How the participants deal with this uncertainty determines their degree of risk aversion. Studies by Hanson and Menezes (1968), Baron (1972) and McAfee and McMillan (1987) confirm that varying the degree of risk aversion affects bidders' behavior. The buyer's surplus received by a bidder *i* with valuation v_i bidding b_i is $v_i - b_i$ if she is awarded the item and zero otherwise. To enhance the probability of winning, the risk-averse bidder increases the size of her bid, reducing her surplus and increasing the seller's surplus (Das & Sundaram, 1997). If either the seller or buyer is risk-averse, the seller prefers the Dutch or first-price auction (Harris & Raviv, 1981; Holt, 1979; Milgrom & Weber, 1982).

When bidders are asymmetric (valuations are no longer based on a common distribution F), the Revenue Equivalence Theory does not hold (Das & Sundaram, 1997; McAfee & McMillan, 1987). Competition from the now disparate bidders has an effect on the determination of bids for the first price sealed bid auction since its bidding strategy

considers both the individual's value and that of the second highest competitor. Within this environment two bidders valuing the item equally may evaluate their nearest rival's value for an item differently, thereby leading to incongruous bids. The inconsistent bidding may award the item to a bidder without the highest value making this auction form inefficient (Das & Sundaram, 1997; McAfee & McMillan, 1987). Auction mechanisms with bidding strategies dependent on the participant's individual valuation (i.e., English, Dutch and Vickrey) remain efficient.

The benchmark model assumes independent private values. Relaxing this assumption gives rise to another extreme, the common value model. Here bidders guess an item's unique true value (McAfee & McMillan, 1987). A phenomenon inherent with the common value assumption is the "winner's curse;" defined by Engelbrecht-Wiggans (1980 pp. 133) as "when the individual to whom the object is awarded tends to be the one who most overestimated the true value of the object." Information asymmetries account for the variation in bids. Here, Milgrom and Weber (1982) show that the English auction performs best in generating the greatest revenue followed by the second-price auction. Dutch and first-price auctions are equivalent as least effective under the common value assumption. Value uncertain bidders can acquire additional information by observing the behavior of other bidders in the English auction. Additionally, Milgrom and Weber (1982) propose that the seller can increase his expected revenue by providing the bidders with information correlated to the item's true value. This phenomon is refered to as the linkage principle is based on the fact that with additional information initially low-value bidders will raise their estimates, thereby promoting more aggressive bidding (Milgrom

& Weber, 1982). More recent literature has found that this principle does not hold beyond single item auctions (Perry & Reny, 1999)

So far we have based bidder payments entirely on the bids. Relaxing this assumption the seller can obtain additional information about valuations (McAfee & McMillan, 1987). There are many auctions, such as for book publishing and mineral rights, where payment depends on both the bid and information revealed *ex post* (via royalty rate or sharing parameter) (Das & Sundaram, 1997). The price paid p is a combination of the bid price, a royalty rate r, and the value $\widetilde{v}$ of the object unknown at the time of the auction $p = b + r\widetilde{v}$. Introducing a royalty rate reduces the impact of the inherent variances in bidder valuations, thus inducing bidders to act more aggressively. Seller revenue rises with aggressive bidding (McAfee and McMillian, 1987).

Most auction theory literature assumes that bidders act non-cooperatively, that is they do not agree to modify their competitive bidding behavior to manipulate equilibrium pricing (Kagel, 1995). In reality, collusion exists in the form of cartels or rings; agreements between bidders regarding bidding aggressiveness and/or predetermination of winners (McAfee & McMillian, 1987). Mead (1967) hypothesized that ascending-bid auctions were more susceptible to collusion than were sealed-bid auctions. Milgrom (1987) confirmed these results using the case of two bidders agreeing to alternate wins. The intuition for this result rests with being able to hide secret price concessions using a sealed bid mechanism, which is impossible in an open ascending auction. Bidding rings or cartels operate on the premise that without competition from the other ring members a designated member can obtain the item at a reduced price. The item is then re-auctioned

among the cartel members, with members sharing in the proceeds resulting from the difference in original auction price and the cartel auction price (McAfee and McMillan, 1987). To combat these activities, Cassady (1967) recommends establishing a reservation price that increases with the number of potential cartel members.

Another consideration for auction models is the potential strength of bidding competition. Holt (1979) and Harris and Raviv (1981) have shown that increasing the number of bidders increases seller revenue on average. They propose that the greater the number of bidders, the smaller the gap, on average, between the value of the highest and second highest bidder (the winning price). In independent private valuation first price auctions the uncertainty of the number of participants can be exploited. McAfee and McMillan (1987) show that if the number of bidders is unknown and they have constant or decreasing absolute risk aversion, then concealing the number of bidders enhances revenue.

The results so far have applied to auctions for a single item. The impact of varying the number of items for auction is gaining a great deal of attention. Both Vickery (1961) and Weber (1983) look at how the "benchmark model" holds in this setting. They discovered that using the Independent Private Values model (IPV) and Nash equilibrium bidding strategies, sustains the Revenue Equivalence when bidders take only one item. Kim (1996) and Lengwiler (1999) look at the issues raised when auctioning more than one item. When the bidder can purchase more than one unit, Engelbrecht and Kahn (1998) claim striking differences emerge. Namely, although a weak form of revenue equivalence holds, the various auction formats (i.e., uniform price, discriminatory or pay-your-bid, and Vickrey) allocate units differently. Attempts have been made to establish

which auction performs better for the seller (Back and Zender, 1993; Noussair, 1995; Katzman, 1995; Engelbrecht-Wiggans & Kahn, 1998). Engelbrecht-Wiggans and Kahn (1998) discovered that with uniform price auctions (a common form of simultaneous multi-unit auction in which high bids win the units, but all units are sold for the same price) tend to encourage zero bids. Pay-your-bid auctions, requiring the winners to pay the individual winning bid, outperform uniform price auctions according to Katzman (1995) while the Vickrey auction provides the seller with the most revenue. The complexity of multi-unit auctions has also affected the study of mechanism optimality. Armstrong (1999) considers the optimality of heterogeneous multiple unit auctions with package bids. Although his analysis was limited to two objects, he found that bundled auctions are efficient but generate different revenue which are strictly optimal in some circumstances. Revenue equivalence does not generally hold within this environment if values are discretely distributed (Armstrong, 1999).

The presence of financial constraints introduces important differences into traditional auction theory. Most notably the revenue equivalence theorem fails (Pitchik & Shotter, 1988; Che & Gale, 1996, 1998). In the case of private information and absolute spending limits, Che and Gale (1996, 1998) found that first-price auctions yield higher expected revenue and social surplus than the other standard auction forms. Also a subsequent bidder's payoff is influenced by the price paid by rival bidders (Benoît & Krishna, 1998; Pitchik & Shotter, 1988). When faced with budget-constrained bidders, the order that items are presented for sale in a sequential auction is important. Benoît and Krishna (1998) demonstrated that it is not always optimal to sell the more valuable object first. Budget constraints likewise impact the strategic bidding behavior of auction

participants. Pitchik and Schotter's 1988 experiments using sequential auction with complete information discovered that the trembling hand perfect equilibrium is more representative than Nash equilibrium in predicting prices. Trembling hand perfection allows for bidders to "tremble" or make mistakes but eventually equilibrium will be achieved by taking their rival's mistakes into consideration in the limit or in this case at the completion of a sequence of auctions (Fudenberg & Tirole, 1991). This equilibrium is robust enough to compensate for the possibility that some players may not play their dominant strategies (Rasmussen, 1989). Financial constraints may be absolute with a preset upper bound, be limited to a certain amount on average, or may be determined endogenously as part of the strategic auction decisions (Engelbrecht-Wiggans, 1987; Benoît & Krishna, 1998). To combat the negative effects of budget constraints Che and Gale (1996) propose a policy that alleviates indivisibility of the good, allows joint bidding, and offers seller provided financing.

Most of what we know about combinatorial auctions has been discovered through empirical studies. Recent experimental work by DeMartini et al. (1999, pp. 22) comparing multi-object auctions reveals "the option to bid for packages clearly improves performance in difficult environments, and does not degrade performance in simple environments." They also presented evidence that the RAD combinatorial auction outperforms both non-combinatorial models and earlier combinatorial (i.e., AUSM) models. Performance was judged on efficiency, auction length, and bidder losses. From the seller's perspective, package bidding was shown to reduce revenues as a percent of the maximum possible (but not average seller revenue). An interesting caveat of their investigation looked at the tradeoff between bidder profits and seller revenue and

suggested that high seller revenue is driven by high bidder losses (DeMartini et al., 1999). This posses two concerns, that of possible bidder default when faced with copious expenses as well as potentially decreasing consumer goodwill. The RAD design was able to achieve revenue gains while at the same time yielding high seller revenue, implying the mechanism is effective in minimizing the tradeoff between the opposing objectives (DeMartini et al., 1999). Solving a large problem with this mechanism could prove computationally intractable requiring some sort of heuristic to reach an acceptable conclusion (Rothkopf et al., 1998). Recently two-sided versions of these combinatorial auctions have been deployed, one for trading environmental emissions permits and another to support bond trading. The bond trading mechanism was built to handle 2,000 bonds (commodities) and 50,000 bids. Given the complex bids allowed, a lot of non-convexities, this means about 200,000 variables and 300,000 constraints. A fully relaxed linear program solution takes about 20 minutes to solve. The heuristic algorithm produces a solution which exceeds 85% of the best known bound, 90% of the time (Personal correspondence Ledyard, 1999). This is the first large-scale introduction and will prove an interesting test of the tractability of the auction form.

2.6 Electronic Auctions

The introduction of auctions on the Internet has heralded a resurgence in their popularity as a selling mechanism and as an area of academic research. Much of the classic auction theory is being reevaluated in light of the new medium and plentiful data. An empirical investigation by Bapna, Goes and Gupta (1998) revealed that classical assumptions might not hold with electronic auctions. Namely, they found heterogeneity

among bidders, characterized by different bidding motivations such as the entertainment value of participation and other rational and irrational bidding strategies. Introduction of software agents, or automated bidding mechanisms, described as "hyper-rational" by Varian (1995), may account for some of this heterogeneity. Yet another study suggests the revenue equivalence theorem is not supported for on-line auctions (Lucking-Reiley, 1999). A field study of 100 on-line auctions by Beam and Segev (1998) recommends a set of criteria for "good" auctions and provides an overview of current practices.

Traditional auctions differ from electronic auctions in several ways that may account for the departure from classical theory. For example, a traditional auction is held at a physical location, is conducted by an auctioneer and lasts a few minutes. In the classical setting this leads to a great deal of expense to establish a site, employ an auctioneer and gather the potential customers. Goods must be transported to a central location and may not be easily examined due to time or physical limitations (Turban, 1997). Conversely, electronic auctions close on average once per week, with many closing daily or hourly, they can be conducted anywhere, use an electronic agent as the auctioneer, and multimedia and database facilities allow for extended complexity of the trade object description (Beam and Segev, 1998; Klein and O'Keefe, 1998). Additionally, the Internet provides a global pool of potential bidders suggesting more aggressive bidding resulting from increased participation. Electronic auctions have lowered entry barriers for all auction participants including auctioneers, suppliers or sellers and consumers (Klein & O'Keefe, 1998). For example, E-Bay is an on-line auction that provides a forum where any seller can submit items for sale and reap the benefits of worldwide exposure. An opportunity associated with electronic auctions is the

potential to establish more complex trading rules through utilization of the environment's computing power (Klein & O'Keefe, 1998).

One of the most noteworthy departures from the classical auction format is the emergence of various multiple object auctions. The strategy of distributing the multiple objects amongst winning bidders and preferences given to those who bid in bulk have formed different on-line auction mechanisms. For example, one of the most popular on-line auction forms is the multi-unit English auction. A common version of this auction type is the "Yankee Auction" in which a specified number of identical items are offered for sale simultaneously. At the close of the auction, the highest bidders win and pay their bid price (Vakrat & Seidmann, 1998). Bids are ranked in order of price, then quantity, then time of initial bid. Specifically, if two or more bids are for the same price, the larger quantity bids take precedence over smaller quantity bids, while if bids and quantity are the same, then the earlier bid takes precedence over later bids (Beam & Segev, 1998). Bapna, Goes and Gupta (1998) describe a modification of the Vickery Auction used on-line which adopts a uniform pricing scheme over the collection of items where each winner pays a price equal to the highest rejected bid.

Modeling the emerging on-line auction forms has proven a formidable task. Vakrat and Seidmann (1998) establish a model that incorporates the "time dimension" of online auctions or the impact of extending bidding over extended periods of time and space. Beam et al. (1999), using a Markov Chain, were able to model a typical online single item auction and extend their model to allow for the sale of multiple identical items where each bidder wants at most one item. The lack of adherence to classical auction theory assumptions and the variety of online mechanisms have hampered the quest for a

single concise model. The most recent advances, combinatorial auctions, have only been investigated experimentally and due to the heuristic nature of their solutions have yet to be modeled definitively.

2.7 Summary

In this chapter we presented a synopsis of classical and emerging auction theory. A great deal of scholarly research exists in the field providing well established guidelines on everything from bidding strategies to optimality of various auction forms. New discoveries are emerging due to the changing environment heralded by auctions migrating to the Internet and the evolution propagated by harnessing computing power. Unique, previously unworkable, auction forms have been introduced that do not readily conform to classical theory therefore revitalizing this exciting area of study.

CHAPTER 3
APPLICATION ENVIRONMENT AND MODEL

3.1 Network Television Practices

Television advertising sales provides an interesting arena in which to investigate the use of the Incompletely Specified Combinatorial Auction (ISCA). Sissors and Bumba (1989) describe network television as a negotiated medium; similar to commodities bought and sold on the commodities exchange. There are three markets for television, "up-front "or long-term, "scatter" or short term and "opportunistic" or last minute buys (Katz, 1995). The majority of sales are conducted during "up-front" where contracts usually involve campaigns spanning an entire broadcast year. Scatter buys are generally for an upcoming quarter while the sale of excess last minute inventory is referred to as opportunistic buys (Sissors & Bumba, 1989).

Advertisers seek to maximize the number of exposures to their desired demographic group per dollar or to minimize the cost per thousand viewers (CPM) while networks attempt to maximize the dollars received per show. Currently all sales are negotiated with no fixed price for placement in individual shows and evaluated on the CPM for a single designated demographic category. Media buyers must meet a predetermined amount of exposures to satisfy campaign goals. Supply and demand plays a significant role in negotiations for what can be considered a perishable good, since any

commercial time that is unsold at airtime can never be recovered. Advertiser demand for show placement, viewer demand indicated by audience delivery estimates, network overhead and expenses, and the proximity to airdate combine to form the basis for network reservation prices (Sissors & Bumba, 1989).

Heterogeneity in campaign length and size, the advertised brand's buying power and estimated show ratings determine an advertiser's valuation for individual units and collectively entire campaigns. These valuations can vary greatly, as do their budgets. Auctions, due to their ability to facilitate price discovery and maximize seller surplus for items with widely dispersed values, are logical sales mechanisms for this environment. The combinatorial auction with its ability to accept package bids is best suited, among the current auction designs, to accommodate the synergies between products as well as the need to aggregate or bundle goods to meet buyer's campaign exposure requirements. In this environment there are also ample substitutes so buyers are not restricted to obtaining a specific item but may be satisfied with any number of substitutes available. The current versions of combinatorial auctions do not allow for substitutes or bids that do not precisely specify the desired objects.

Our auction will be employed to accommodate the following "up-front" sales practices. Commercial sales are negotiated by daypart, i.e., specific time slots within the day such as primetime, daytime, kids and sports. We will concentrate on primetime where approximately 250 to 300 buyers compete for a finite amount commercial airtime in the upcoming season. Primetime extends from 8 p.m. to 11 p.m. with shows varying

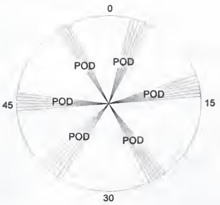


Figure 3.1 Example of Pod Placement in an Hour Show

in length from 30 minutes to 2 hours. An hour show generally contains 5 to 7 commercial pods and roughly 4 to 8 15-second slots per pod. See Figure 3.1. A pod is a collection of commercials normally lasting one to two minutes and includes a number of commercials of various lengths. The base unit in our auction will be the 15-second spot.

All national television time is priced based on a 30-second spot. Advertisers wishing to utilize longer or shorter duration commercials (i.e., 15 second or 60 second) can expect the amount charged to be adjusted according to the length (Katz, 1995). Rarely, networks will charge a premium for handling of non-standard commercial lengths to discourage a large number of 15-second commercials that contribute to clutter. Clutter, generated by an overwhelming number of ads appearing in a show or pod, dilutes the strength of the advertiser's message (Sissors & Bumba, 1989). Our model assumes the price for a 15-second (60-second) commercial is one-half (twice) that of a 30. Campaigns can consist of either a single length commercial or a mix of lengths. For example, an advertiser may run strictly 15, 30, 45 or 60-second commercials, or any combination of these lengths in a single campaign.

Although it would appear most cost effective to purchase all 15-second commercials, since reach, or the number of viewers exposed to a commercial, is relatively the same regardless of commercial length, the 30-second commercial is most prevalent. It has been suggested that 30-second commercials better satisfy the creative needs of advertisers who are seeking to gain both consumer attention and convey the product message.

To allow comparison between packages of various length commercials, advertisers use 30-second equivalents, where a 15-second commercial represents a half a unit, with 30-seconds being the base unit and a 60-second commercial is comparable to 2 units. We simplify our calculations by redefining the base unit as a 15-second commercial representing 1/2 the reported demographics and the longer commercials a multiple of this new base unit. This corresponds with our treatment of unit pricing. Although, as noted earlier, the amount of exposure is not effected by the length of commercial, in this scenario the demographics are scaled by the length of the commercial. The show's list price is then divided by the total number of exposures to determine the Cost Per Million (CPM). This equivalized CPM allows the buyer to analyze the best mixture of commercial lengths.

3.1.1 Environmental Constraints

There are constraints imposed on the placement of ads. It is common practice to guarantee that competing products do not appear in the same pod, referred to as "pod protection." Two similar products can advertise in the same show but every effort is

made to ensure that they do not appear in the same commercial break. However, 15-second commercials are excluded from this protection.

Media buyers often express preferences for placement in particular shows and occasionally sales are conditional on acquiring those specified shows. In addition, buyers may require that their advertisements not appear in selected shows due to what may be deemed inappropriate content.

15-second commercials require special handling. To avoid clutter, industry practice allows at most two 15's in the same pod. Duplicate ads, or more than one advertisement of any length from any one buyer, are not allowed to appear in the same pod. However, this restriction is relaxed for 15-second commercials. If, for example, an additional 15-second spot is required to complete a pod then a single advertiser's 15's may be "book-ended" or placed at the beginning and end of the same pod.

Airdates are also a critical consideration. Although, advertisers may not have specific tastes for individual programs, they may require that an ad appear on certain dates to coincide with other media campaigns (i.e., radio, billboard, cable television, etc). Campaigns are scheduled as continuity (continuous over a length of time), bursts (ads placed at a specific frequency over an extended period such as twice a month all year), or flights where ads aired for specific periods are followed by periods of inactivity (Katz, 1995).

3.1.2 Negotiation Strategies

Individual show pricing and commercial availability statistics are considered proprietary and jealously protected by both the network and advertisers. Although

television stations may publish "rate cards," the rates shown are viewed as the starting point for negotiations and do not reflect the ultimate prices (Merskin, 1999). Both parties exploit these information asymmetries during mediation. There is considerable negotiation back and forth in terms of what the media buyer is willing to pay for a particular offering or collection of show placements and what the network representatives feel is a fair and acceptable price. In this industry there is no after market where buyers can sell directly to other advertisers. Network representatives calculate their package prices based on a discount rate to the list price. This rate varies with supply and demand but has an explicit upper limit established by the daypart manager.

A variety of facts are exchanged between parties in the negotiation. They usually include the buyer's stated budget (not necessarily his true budget), a minimum reach requirement for designated demographic, the length(s) of commercials for the campaign. Flighting information is also provided which often, but not always includes, a set of desired and/or forbidden shows or air-dates, the length of commercials allowed in each show and a maximum number of commercials per show. The seller also knows the type or brand of service/product advertised. Collectively, the information provided allows the network to generate a package that is presented to the media buyer for approval. The deal is evaluated based on a 30-second equivalized cost per million (CPM) for the demographic. Proposals are iteratively modified until the parties reach an agreement. Our auction model accepts multi-dimensional bids from all buyers and allows market forces to determine the allocation and prices that generate equilibrium.

3.2 Auction Description

We propose a progressive semi-closed auction format that allows the media buyer to dynamically create individual bundles from a selection of commercial slots upon which they then bid. Our research will provide a new and innovative way of using an auction mechanism by allowing inexact bidding with multiple evaluative criteria as well as providing for constraints unique to the television sales environment. Bidders are given the flexibility to change and or modify their bids and bundles until a stopping criterion has been reached. Suggestions are provided to the buyers to help them formulate successive bids, but active pricing will not be disclosed. This semi-closed format, proposed by Tiech (1999), will satisfy the need for non-disclosure of market prices that is required by both buyer and seller. Additional constraints that will be modeled include separation of competing ads, meeting bidder's demographic group exposure requirements while ensuring the seller receives his reservation prices, accommodating specific show placement (non-placement) requests with commercial length specification while not exceeding an upper bound on the number of ads allowed per show. Our model assumes that campaigns are continuous throughout the season thus does not provide for flighting nor does it contain provisions for special programming that may displace regularly scheduled shows during the course of the year.

We model our auction as an integer program. A summary of the notation is presented in Table 3.1. Our main decision variable is $x_{u,p,s,b}$, a binary variable set to 1 if a particular buyer b is allotted a unit u in pod p for show s.

3.2.1 Notation

Table 3.1 Summary of Notation

General	
u,p,s,b,i	Subscripts *s*=show, *b*=buyer, *p*=pods, *u*=pod part , *i*= allowable ad campaign length (using this order).
DV	Signifies a decision variable.
Shows	
S	Number of shows.
P_s	Number of pods in show *s*.
L_s	List price for each 15-second unit in show *s*.
D_s	Vector of 15-second demographic values for show *s*.
$U_{p,s}$	Number of 15-second portions in pod *p* for show *s*.
C_s	Maximum number of total units in show *s* available to sell.
Buyers:	
B	Number of buyers.
T_b	Target Vector of desired demographic impressions for buyer *b*.
h_b	Vector of desired shows for buyer *b* ($h_{s,b} = 1$ if show s is desired by buyer b, 0 otherwise) Note that h_b can be a zero vector.
$N_{s,b}$	Set of specified commercial length(s) in show *s* for buyer *b*.
$\overline{N}_{s,b}$	Set of allowable commercial length(s) in show *s* for buyer *b*.
$\underline{H}_b, \overline{H}_b$	Min/max number of desired shows that buyer *b* *must* have ($\underline{H}_b \leq h_b \leq \overline{H}_b$).
$K_{s,b}$	Number of correct length commercials allowed in show *s* by buyer *b*.
M_b	Type of merchandise advertised by buyer *b*.
$a_b, h_b, \underline{H}_b, \overline{H}_b, I_{p,s,b}, N_{s,b}, M_b, K_{s,b}$	bid from buyer *b*. a_b is the amount bid.
$v_b(d)$	value of buyer *b*'s advertising given a cumulative demographic vector *d*.
Seller:	
$f_{p,s,b}$	**DV:** 1 if bidder *b* has more than 15-seconds in pod *p* in show *s*.
y_b	**DV:** 0,1 variable. If 0, buyer *b* can't buy any pods *p*.
$x_{u,p,s,b}$	**DV:** 0,1 variable. If 1, buyer *b*, has spot u in pod *p* in show *s*.
$I_{p,s,b,i}$	**DV:** 0,1 variable. If 1 pod *p* of show *s* for buyer *b* uses an allowable number of advertising slots.
$z_{p,s,b}$	**DV:** 0,1 variable. 1 if $\sum_{i \in \overline{N}_{s,b}} I_{p,s,b,i} = 1$ otherwise 0. (notational simplicity)
$j_{s,b}$	**DV:** 0,1 variable. If 1, buyer *b* has any unit in show *s*.
r	budget discount rate applied to the list price.

3.2.2 Objective Function

This problem focuses on a fixed time horizon of some specified number of weeks, each week of which is a repeat of the pattern sold in the auction. The seller solves

$$\text{P1: } \max \sum_{b=1}^{B} a_b y_b$$

The objective function maximizes the total revenue from accepted bids, a_b. The variable y_b is an indicator variable that is set to 1 if the bid is accepted subject to the following constraints.

The objective in this case is to maximize revenue. This approach is selected, over maximizing profit because it achieves a stated goal of satisfying a predefined budget. Additionally, the product involved is considered perishable and therefore the seller is more concerned with depleting inventory than selling at the greatest profit.

Should profit be the motivating factor our objective function can be restated as follows:

$$\text{(AP1) } \max \sum_{b=1}^{B} a_b y_b - (1-r)\left(\sum_{b=1}^{B}\left(\sum_{s=1}^{S}\left(\sum_{p=1}^{P_s}\sum_{u=1}^{U_{p,s}} x_{u,p,s,b}\right)L_s\right)\right)$$

With this formulation the seller reservation price is incorporated into the objective and therefore can be dropped from the constraints, i.e. Equation (1.1).

3.2.3 Constraints

Rarely are sales predicated solely on price and availability. Incorporating the constraints of the environment may be more challenging than determining the highest

bids in the television industry. The following constraints are required to adequately represent the restrictions inherent in commercial airtime sales.

3.2.3.1 Reservation requirement

The reservation requirement in the television industry is based on a discount rate, r that is applied to the list price for each show, L_s. Daypart managers must meet an annual budget that is a proportion of the total possible revenue available based on the list price. The discount to list reflects this proportion, however, discount rates for individual sales may be above or below the discount as long as ultimately the aggregate meets or exceeds the budget. Therefore the sum of the accepted bids must be greater than the discounted commercials purchased where $x_{u,p,s,b} = 1$ indicates that a commercial has been placed in pod p of show s by buyer b. The following formula first determines the non-discounted revenue per show over all the shows and buyers and then applies the discount rate. The resulting discounted revenue requirement is compared with the total amount bid by all buyers to insure that the minimum reservation price is received.

$$(1.1) \quad \sum_{b=1}^{B} a_b y_b \geq (1-r)\left(\sum_{b=1}^{B}\left(\sum_{s=1}^{S}\left(\sum_{p=1}^{P_s}\sum_{u=1}^{U_{p,s}} x_{u,p,s,b}\right)L_s\right)\right) \qquad \textbf{reservation requirement}$$

3.2.3.2 Maximum seller coverage

Although there is a finite amount of commercial airtime available, not all is targeted for sale during "up-front" sales. The maximum coverage constraint insures that at most a specific number of commercials are sold per show as noted by C_s.

$$(1.2) \quad \sum_{b=1}^{B}\sum_{p=1}^{P_s}\sum_{u=1}^{U_{p,s}} x_{u,p,s,b} \leq C_s \quad s = 1,\ldots,S \qquad \textbf{maximum coverage}$$

3.2.3.3 Maximum spot availability

Each show is broken into pods, usually one- to two-minute blocks of airtime reserved for commercial placement. The number of pods per show and their length vary from show to show. The following guarantees that the number of commercial placements per pod does not exceed the number of spots available to accommodate them. The number of individual units in each pod summed over all buyers must be less than or equal to the total number available in that pod.

(1.3) $\sum_{b=1}^{B}\sum_{u=1}^{U_{p,s}} x_{u,p,s,b} \le U_{p,s} \quad p=1,...P_s \;\; s=1,...,S$ **max availability per pod**

3.2.3.4 Buyer selection indicator

If a particular bidder b does not obtain any airtime the following constraint forces all his x values to zero and thus drops her bid from consideration.

(1.4) $\sum_{s=1}^{S}\sum_{p=1}^{P_s}\sum_{u=1}^{U_{p,s}} x_{u,p,s,b} \le \left(\sum_{s=1}^{S}\sum_{p=1}^{P_s} U_{p,s}\right) y_b \quad b=1,...,B$ **can't buy if not selected**

Equation (1.4) can also be written as

(A1.4) $x_{u,p,s,b} \le y_b \;\; u=1,...,U_{p,s} \;\; p=1,...,P_s \;\; s=1,...,S \;\; b=1,...,B$

This alternative formulation is an example of constraint disaggregation for binary variables and has been shown to provide for a stronger LP relaxation (Johnson, Nemhauser & Savelsbergh, 2000). The original formulation will cause B constraints to be included in the formulation, one for each bidder in the problem. The alternative expression enumerates each combination of bidder, show, pod and unit. Adding more constraints and thus growing the size of the problem seems counterintuitive to the goal of improving the LP relaxation. However, Johnson, et al. (2000, pg. 5) suggest that "to

obtain strong bounds, it may be necessary to have a formulation in which either the number of constraints or the number of variables (or possibly both) is exponential in the size of the natural description of the problem."

3.2.3.5 Campaign commercial length constraint

Advertising campaigns may consist of commercials of varying lengths. A campaign composed of only 15, 30, 45 or 60-second spots must eliminate any collection of 15-second units in an individual pod that will not form the desired commercial length. Buyers may also have mixed campaigns, or campaigns that consist of a combination of lengths. $N_{s,b}$ is a set of permissible commercial lengths for each show s supplied by buyer b. Note that the permissible collection of lengths can vary by show facilitating a buyer's need to change the length(s) of the campaign over time. Industry practice dictates that no more than one commercial per buyer appears in the same pod. An exception to this rule allows that at most two 15-second units from the same advertiser may be placed in the same pod to fill an empty 15-second slot and complete the pod. To account for this exception we define

$$\overline{N}_{s,b} = \begin{Bmatrix} N_{s,b} \cup \{2\} \text{if } 1 \in N_{s,b} \text{ and } 2 \notin N_{s,b} \\ N_{s,b} \text{ otherwise} \end{Bmatrix}.$$

Together Equations (1.5a) and (1.5b) require the selected units in each pod to correspond to one of the allowable lengths listed in $\overline{N}_{s,b}$. The unit slots are numbered for convenience but the number does not correspond to a specific location in a pod, therefore there is no need to insure that the units are consecutive when forming a 30, 45 or 60-second commercial.

(1.5a) $\sum_{u=1}^{U_{p,s}} x_{u,p,s,b} = \sum_{i \in N_{s,b}} i I_{p,s,b,i} \quad p = 1,\ldots,P_s \;\; s = 1,\ldots,S \;\; b = 1,\ldots,B$ **campaign length**

Equation (1.5b) prevents more than one correct length commercial for buyer *b* from appearing in pod *p* of show *s*. Note, two 15-second commercials are allowed in one pod if $1 \in N_{s,b}$. Note that $z_{p,s,b}$ is equivalent to $\sum_{i \in N_{s,b}} I_{p,s,b,i}$ indicating if set to 1 that buyer *b* has a correct length commercial in pod *p* of show *s*. $z_{p,s,b}$ is used within our formulation to simplify the notation but will not appear as a variable in the implementation of our problem.

(1.5b) $z_{p,s,b} \le 1 \quad p = 1,\ldots,P_s \;\; s = 1,\ldots,S \;\; b = 1,\ldots,B$ **commercials per pod**

3.2.3.6 Anti-Clutter Control

Placing a large number of different commercials in the same pod weakens the impact of all commercial messages within that pod. This phenomenon results from the "clutter" exacerbated by the use of 15-second commercials. To reduce clutter, Equation (1.6) allows at most two 15-second ads to appear in each pod.

(1.6) $\sum_{b=1}^{B} I_{p,s,b,1} \le 2 \quad p = 1,\ldots,P_s \;\; s = 1,\ldots,S$ **anti-clutter**

3.2.3.7 Frequency: Max commercials per show

Controlling the number of commercials appearing in each show will provide the buyer with the ability to spread or aggregate commercials over the length of a campaign week. $K_{s,b}$ indicates the number of correct length commercials that are allowed in show *s* by buyer *b*. Should a buyer want to place all of her inventory early in the week she would set $K_{s,b}$ high for shows during the desired days and other days to zero. A buyer can

identify a forbidden show, say show s', by setting $K_{s',b} = 0$. Should a buyer forbid a show, during implementation of the model, the corresponding variables will not be generated. Normally, buyers want their ads to appear only once per show to enhance the number of different viewers that are exposed to their spot, in this case all $K_{s,b}$'s would be set to 1. Equation (1.7) provides for this constraint.

$$(1.7) \qquad \sum_{p=1}^{P_s} z_{p,s,b} \le K_{s,b} \quad s = 1,\dots,S \;\; b = 1,\dots,B \qquad \textbf{maximum spots per show}$$

3.2.3.8 Demographic gross impression guarantee

Media buyers desire a specific amount of demographic reach, or number of people exposed to their commercial during their campaign. There are a variety of demographic categories upon which a show is rated. Each show's gross impressions per category forms the demographic vector D_s and indicates the seller's estimated reach for that particular show in the upcoming season. D_s is ordered by the categories: Women 18 to 49, Women 25-54, Men 18-49, Men 25-54, Adults 18-49 and Adults 25-54. T_b represents the vector of demographic reach or gross impressions that the buyer needs to meet the product's campaign goals and is ordered with the same categories in D_s. Note that although total number of gross impressions per show, D_s, in reality does not change with the length of the commercial our model uses equivalized 30-second calculations that differentiate based on the length of commercial. For example, the number of gross impressions for a 15-second spot is 1 unit of demographics while the unit gross impressions for a 30, 45 or 60-second commercial is 2, 3 and 4 respectively. Bidders normally evaluate their package on a single demographic category. The sum of the total

number of 15 second units, times the specified demographic gross impressions over all selected shows must meet or exceed the required reach for the specified demographic group(s). Equation (1.8) assures that the minimum demographic requirement is achieved.

$$(1.8)\quad \sum_{s=1}^{S}\left(\sum_{p=1}^{P_s}\sum_{u=1}^{U_{p,s}} x_{u,p,s,b}\right)D_s \geq T_b y_b \quad b=1,\dots,B \qquad \textbf{demographic reach required}$$

3.2.3.9 Show placement requirement

In addition to the actual dollar amount bid and demographic requirements, a buyer may specify desired shows within which they would like their commercials placed. Setting $h_{s,b}$ to 1 indicates that buyer *b* wants placement in show *s*. She can further indicate her willingness to deviate from her program choice by setting the upper $\overline{H}_b$ and lower $\underline{H}_b$ bounds to the number of shows required. $j_{s,b}$ identifies which shows a buyer has been allocated and is determined with the following formulas. $j_{s,b}$'s is set equal to 1 if buyer *b* has any slot in show *s* and 0 otherwise.

$$(1.9a)\quad \sum_{p=1}^{P_s} z_{p,s,b} \geq j_{s,b} \quad s=1,\dots,S \;\; b=1,\dots,B$$

Show allocation

$$(1.9b)\quad \sum_{p=1}^{P_s} z_{p,s,b} \leq P_s j_{s,b} \quad s=1,\dots,S \;\; b=1,\dots,B$$

Equation (1.10) ensures that a buyer is allotted at least $\underline{H}_b$ shows and no more than $\overline{H}_b$ of the shows requested. To satisfy this constraint at least $\underline{H}_b$ $j_{s,b}$'s will have to be set to 1 and no more than $\overline{H}_b$.

$$(1.10)\quad \overline{H}_b \geq \sum_{s=1}^{S} h_{s,b} j_{s,b} \geq \underline{H}_b y_b \quad b=1,\dots,B \qquad \textbf{desired shows}$$

3.2.3.10 Pod protection constraints

Networks routinely guarantee that competing advertisements do not appear in the same pod. The group of equations (1.11 a-d) implements this notion of "pod protection." Pod protection is normally not given to 15-second commercials, therefore we need only investigate anti-competition when a buyer has two or more units in a particular pod. The decision variable $f_{p,s,b}$ is set to 1 if a bidder *b* has two or more 15-second units in a particular pod *p* of show *s*. When the number of units a bidder has per show is 0 there is no competition and Equation (1.11a) forces both $f_{p,s,b}$ and $z_{p,s,b}$ to zero.

$$\text{(1.11a)} \sum_{u=1}^{U_{p,s}} x_{u,p,s,b} \geq z_{p,s,b} + f_{p,s,b} \quad p=1,\ldots,P_s \;\; s=1,\ldots,S \;\; b=1,\ldots,B\,,$$

In the case where one unit is assigned in a particular show to buyer *b,* pod protection is not enforced and the requirement that $z_{p,s,b}$ equal or exceed $f_{p,s,b}$ in Equation (1.11b) sets $f_{p,s,b}$ to zero, and $z_{p,s,b}$ to at most one. This corresponds with the fact that buyer *b* has a single unit in any pod of show *s*.

$$\text{(1.11b)}\; f_{p,s,b} \leq z_{p,s,b} \quad p=1,\ldots,P_s \;\; s=1,\ldots,S \;\; b=1,\ldots,B\,,$$

Equation (1.11c) will force $z_{p,s,b}$ to one in this case.

Pod protection only becomes an issue when two or more units are assigned within the same pod of a show to a single bidder thereby generating a potential 30-second or longer commercial. Equations (1.11a-c) will force $z_{p,s,b}$ to one and $f_{p,s,b}$ to one when two or more spots are bought by buyer b in show s, pod p.

$$\text{(1.11c)} \sum_{u=1}^{U_{p,s}} x_{u,p,s,b} \leq z_{p,s,b} + \left(\sum_{p=1}^{P_s} U_{p,s} - 1 \right) f_{p,s,b} \quad p=1,\ldots,P_s \;\; s=1,\ldots,S \;\; b=1,\ldots,B\,,$$

Finally, Equation (1.11d) will keep competitors away from a protected pod in show s for buyer b.

(1.11d) $f_{p,s,i} + z_{p,s,j} \le 1 \quad i < j, j = 2,...,B \quad \forall M_i = M_j, \quad p = 1,...,P_s \quad s = 1,...,S$.

An example will help clarify this fairly complicated methodology. Suppose we have two buyers i and j who are being considered for placement in pod p of show s. Further suppose that buyer i wants a 30-second spot in the pod and buyer j wants a 15-second commercial in the same pod. Therefore the $\sum_{u=1}^{U_{p,s}} x_{u,p,s,i} = 2$ and $\sum_{u=1}^{U_{p,s}} x_{u,p,s,j} = 1$. Table 3.2 lists the possible values of the z and f variables in equations 1.11a through 1.11c. If buyer

Table 3.2 Variable Value Allocations for Pod Protection

Variable / Equation	30-second or > Commercial		15-Second Commercial		No Commercial	
	$f_{p,s,i}$	$z_{p,s,i}$	$f_{p,s,j}$	$z_{p,s,j}$	$f_{p,s,i}$	$z_{p,s,i}$
1.11a	0 or 1	0 or 1	0 or 1	0 or 1	0	0
1.11b	0 1	0 or 1 1	0	1	0	0
1.11c	1	1	0	1	0	0

i has a 30-second or greater commercial in a pod both $f_{p,s,i}$ and $z_{p,s,i}$ are set to 1 through the series of equations. Buyer j, with a 15-second commercial in the same pod, has only his $z_{p,s,j}$ set to 1. Equation 1.11d uses the fact that only 30-second or greater commercials have both $f_{p,s,b} = 1$ and $z_{p,s,b} = 1$, while 15-second commercials have only $z_{p,s,b} = 1$ to prevent both from being placed in the same pod. If a buyer does not have a commercial in a pod both variables are set to zero. In our example

$(f_{p,s,i}=1)+(z_{p,s,j}=1)>1$ is a contradiction to what is allowed by Equation 1.11d. Therefore, since Equation 1.11d is only applicable when the products advertised are the same between two buyers, buyers i and *j* could not appear in the same pod if they were selling like products.

3.2.3.11 Bid specification and ordering

The action starts with buyers placing bids $(a_b, h_b, \underline{H}_b, \overline{H}_b, I_{p,s,b}, N_{s,b}, M_b, K_{s,b})$. If a buyer submits more than one bid, the most recent one is used. If more than one bid is for the same amount, priority is given to the earlier bid through a lexicographic ordering imposed by altering bids with a timestamp, t_b, as

$$\bar{a}_b \leftarrow a_b + \frac{t-t_b}{M}$$

where t is the current time and M is sufficiently large.

3.2.3.12 Bidder reservation price

The auctioneer solves P1 and presents the solution to the buyers. An infeasible solution may also be announced in which case, nothing is accepted by the seller. A rational buyer will only accept a feasible solution if

$$v_b\left(\sum_{s=1}^{S} D_s\left(\sum_{p=1}^{P_s}\sum_{u=1}^{U_{p,s}} x_{u,p,s,b}\right)\right) \geq a_b,$$
$$\text{where } a_b \leq \text{Bidder's Budget.}$$

If everyone accepts the solution, the auction is over. Otherwise, new bids can be submitted.

When there is only one show and one pod with only one pod part, this reduces to a normal first price English auction.

3.3 Summary

This chapter defines the network television advertising sales environment. The description includes the product characteristics, environmental constraints imposed on the allocation of goods and current negotiation strategies. A detailed integer program was developed to incorporate industry practices into a semi-sealed progressive combinatorial auction designed to replace the current negotiated environment. The problem objective is to maximize seller revenue while satisfying all constraints imposed by both the buyers and the seller. Buyer constraints and requirements are conveyed via a multi-criteria incompletely specified package bid. To accommodate industry practices the auction problem becomes quite complex. A few of the many constraints incorporated in the model include those designed to separate competing commercials, retain a portion of inventory for later markets and achieve specified demographic exposures in a particular demographic category while satisfying individual show placement requests. The complexity and combinatorial nature of the auction suggests the need for a heuristic solution method, which is explored is subsequent chapters.

CHAPTER 4
CONSTRAINT PROGRAMMING

4.1 Introduction

Constraint Satisfaction Problems (CSP) involve finding values for all problem variables that simultaneously satisfy all problem specific constraints. Constraints can be viewed as a relationship between variables that restrict their possible instantiations. The paradigm has been studied since the 1960's and 1970's when the artificial intelligence community applied it to picture processing (Montanari 1970; Waltz 1975). The ability to achieve solutions to these complex problems rests on the notion of eliminating impossible alternatives from consideration as early in the allocation as possible. Early elimination reduces domains from which values are chosen and thus facilitates expeditious results.

Recently a great deal of attention has focused on Constraint Programming (CP) due to its ability to solve combinatorial problems such as the one described in this research. Constraint Programming takes the solutions to a standard CSP and applies them to an objective function which is successfully tightened to find an optimal solution (Bartak, 1999). Constraint Programming has various advantages over other methodologies. Of major importance is the time it takes to achieve a solution. CP algorithms can often achieve solutions more quickly than can integer programming methods. Additionally, CP representation corresponds more closely with the entities of

the original problem. Thus making formulations simpler to compose, heuristics more readily developed and the solutions less difficult to interpret (Bartak, 1999). By using constraint programming as a basis for our heuristic we hope to capitalize on these advantages.

This chapter begins with a brief introduction to constraint satisfaction problems. Section 4.2 defines the constraint satisfaction problem and methodology. Various CSP solution techniques are described in sections 4.3 through 4.6. Section 4.7 introduces Constraint Programming. The remaining section explores how these methodologies apply to the auction problem presented in this research.

4.2 Constraint Satisfaction Problems

The auction problem developed in Section 3.2 is a combinatorial auction and thus achieving an optimal solution has been shown to be an NP-Complete problem (Rothkopf et al., 1998). As such, a heuristic is required to allow a satisficing solution to be reached in real time. Combinatorial problems are found in areas such as planning, scheduling, generalized assignment and resource allocation and have been effectively formulated as constraint satisfaction problems (CSP) (Nonobe & Ibaraki, 1997). Constraint Satisfaction is a general term describing a class of problems involving a set of variables that are to be instantiated from an associated domain while satisfying a set of constraints that limit the assignment (Mackworth, 1992). CSPs use a variety of search methodologies to find a feasible solution.

The goal of a constraint satisfaction problem (CSP) is the assignment of values to its variables that will satisfy all constraints. More formally, the finite CSP is defined by

its three components. V: a finite set of variables $\{X_1, X_2, ..., X_n\}$, D: the set of corresponding domains $\{D_1, D_2, ..., D_n\}$ where D_i is the finite domain of X_i, and C is a finite set of constraints or relations $\{C_1, C_2, ..., C_r\}$ restricting the assignment of values to variables. A constraint $C_{i,j,k...}$ between the variables $X_i, X_j, X_k...$ is any subset of the possible combinations of values of $X_i, X_j, X_k....$ For example the cross product $C_{i,j,k...} \subseteq D_i \times D_j \times D_k \times ...$ indicates the possible combinations of values that the constraint allows (Brailsford, Potts, & Smith, 1998). If there exists an assignment of a value from a variable's domain for all variables that satisfy every constraint then there is a feasible solution to the problem, otherwise it is said to be unsatisfiable.

Solving the CSP can be accomplished by either constructing a solution by iterative variable assignments leading to a feasible solution or by starting with an initial solution (but not necessarily feasible) and subsequently modifying or repairing the solution until it becomes feasible. Known as the systematic search or "constructive" approach, the former methodology applies backtracking techniques and is usually designed to solve a given problem instance exactly. While the stochastic search or "repair" approach gradually repairs an initial solution in order to reduce the infeasibility until all constraints are satisfied. Several greedy algorithms, such as tabu, genetic algorithms or neural networks have been shown to be effective in generating initial solutions. The repair approach has been found to be particularly effective for large-scale problems (Nonobe & Ibaraki, 1997).

4.3 Arc Consistency

In a binary CSP, constraints involve only two variables and are visually depicted by a constraint graph (Figure 4.1). The nodes of the graph represent the variables and the

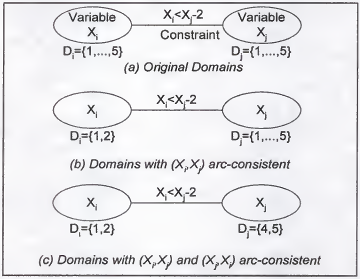


Figure 4.1 Constraint Graph (Brailsford et al., 1999)

lines or arcs connecting the nodes represent the constraints between them. Arc-consistency is achieved by reducing the domains of the problem variables until the remaining values are all supported; a value is supported if every constraint on the variable includes a tuple in which the variable takes this value and all other variables take supported values. For example if a constraint $C_{i,j}$ exists between variables X_i and X_j the arc (X_i, X_j) is arc consistent if for every value a in the domain D_i of variable X_i there is a value b in the domain D_j of variable X_j that satisfies the constraint $C_{i,j}$ (see Figure 4.1b & c). In Figure 1b, (X_i, X_j) is arc-consistent but (X_j, X_i) is not, while in

Figure 4.1c the variables are fully arc-consistent. $b \in D_j$ is called a supporting value for $a \in D_i$. Any values of a that do not satisfy the constraint, i.e. have no supporting value, are removed from the domain D_i and in so doing makes the arc (X_i, X_j) arc-consistent (Brailsford et al., 1999). However, the simple fact that none of the arc-consistent domains are empty does not imply that the CSP has a solution. A solution is achieved only if all variables can be assigned a specific value such that they all support each other.

There are several arc-consistency algorithms employed to establish consistent domains. The importance of the algorithms rests in their ability to reduce the size of the problem and save computational processing time. Arc-consistency has been widely used as a preprocessing step to eliminate local inconsistencies before any attempt is made to construct a solution. Several algorithms have been developed that capitalize on the knowledge about constraint properties to reduce the cost of consistency checking, see Chen (1999) for an overview.

CSPs are frequently a subpart of a larger application. In these cases it is often important to compute all possible solutions which can be systematically explored to find the best configuration for a given situation. Optimization problems can assume this approach, thus delaying optimization criterion development until a set of solutions has been discovered. This technique is tractable when the possible value assignments are discrete but faces challenges with continuous values since continuous domains admit an infinite set of values. To overcome the complexity of continuous value representation in a binary constraint environment Sam-Haroud & Faltings (1996) suggest descretizing variable ranges into one or a small collection of intervals that roughly approximate a

constraint by an enclosing box whose borders represent the unary outer projections of the variables involved. In this case values falling within the confines of this box are considered arc-consistent.

4.4 Systematic Search Algorithms

Systematic search algorithms involve attempting to achieve a consistent solution by repeatedly extending partial solutions. The most basic algorithm called Generate and Test (GT) randomly selects a variable to instantiate and then checks that the labeling is consistent. It is inefficient in that the random selection of variables does not capitalize on problem specific information and thus must perform an exhaustive search. Backtracking improves on this technique and is best described as a depth-first instantiation technique. Another alternative involves enforcing arc-consistency, an elimination approach ruling out all solutions containing local inconsistencies (Mackworth, 1992). A branch and bound type search tree is typically used to graphically represent the current state of the search. A node represents a partial solution and the branches different values that could be assigned to some variable. *Past variables* are those that have already been assigned a value, while a *future variable* has not yet been assigned a value. Choosing a branch of the tree to explore instantiates the variable with the value associated with the chosen branch. Should the domain of a future variable become empty the problem has reached a *dead-end* or has become annihilated. Note that mathematical programmers would use different terminology to describe the elements of the problem. For example, they would say, "fathomed" instead of "dead-end."

4.4.1 Look Back Algorithms

The most common algorithm for performing systematic search is backtracking; an approach that after variable instantiation "looks back" to ensure the assignment is consistent with previously instantiated variables. Backtracking incrementally attempts to reach a complete solution from an intermediate partial solution by repeatedly assigning values consistent with the partial solution. If a consistent value cannot be found the algorithm backs up to a point where successful choices can be made. The method that is used to choose which previous variable to return to in the event of inconsistencies defines the various backtracking algorithms.

4.4.1.1 Chronological Backtracking (BT)

Chronological Backtracking (Bitner & Reingold, 1975) is the generic backtracking algorithm. At every stage of backtracking search, there is some current partial solution that the algorithm attempts to extent to a full solution. The process begins with the current variable being assigned a value from its domain. Then consistency is checked between this instantiation and the instantiations of the current partial solution. If any constraint between this variable and the past variables is violated the assignment is abandoned and the next domain value of the current variable is tried. If there are no more domain values left, BT backtracks to the most recently instantiated past variable, assigns it a new value and the process repeats. If all checks succeed, the branch is extended by instantiating the next variable to each of the values in its domain. If a value has been assigned to every variable a complete solution has been found otherwise the problem is infeasible (Mackworth, 1992).

4.4.1.2 Backjumping (BJ)

Backjumping (Gaschnig, 1977) is similar to, but more intelligent than, Chronological Backtracking. BJ identifies the latest instantiated variable causing a constraint failure and proceeds directly to that variable when it reaches a dead-end. Instead of chronologically backtracking to the preceding variable, BJ jumps back to the

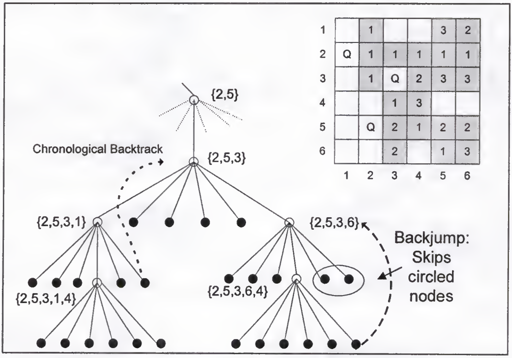


Figure 4.2 Partial Backtrack Tree (Kondrack & van Beek, 1997)

deepest past variable that was checked against the current variable. For example, Figure 4.2 represents a partial backtracking tree from an n-queens problem described by Kondrak and van Beek (1997) that shows how the backjump technique skips the circled nodes. (N-queens is a classic problem used in artificial intelligence to demonstrate difficult problems. The goal is to assign chess queens positions the on an n by n game board such that no queen can capture another.) BJ reduces the number of consistency

checks by skipping search tree nodes thus it behaves more efficiently when all instantiations are inconsistent for the current variable. Changing the value assignment of the failure causing past variable may allow a consistent instantiation to be found for the current variable. Backtracking to any of the intervening variables will have not effect since they have not impact on the reason for the failure.

4.4.1.3 Conflict-Directed Backjumping (CBJ)

By tracking previous failures, Conflict-Directed Backjumping (Prosser, 1993) demonstrates more sophisticated backjumping behavior than BJ. Every variable has its own conflict set that lists the past variables that have failed consistency checks with this current instantiation. Every time a consistency check between the instantiation of the current variable and an instantiation of some past variable fails, the past variable is added to the conflict set of current variable. When all possible values for the current variable have been exhausted, CBJ backjumps to the deepest past variable in its conflict set, this variable becomes the current variable and a new value assignment is attempted. Note that the variables in the conflict set of the variable that could not be instantiated are propagated up the tree and added the conflict set of the past variable so that no conflict information is lost. Figure 4.3 depicts a conflict set that would be formed for this example.

4.4.2 Look Ahead Algorithms

A disadvantage of "Look Back" algorithms is late discovery of conflicts. The "Look Ahead" approach attempts to overcome this problem by looking at future variable assignments and eliminating impossible values from consideration earlier in the process.

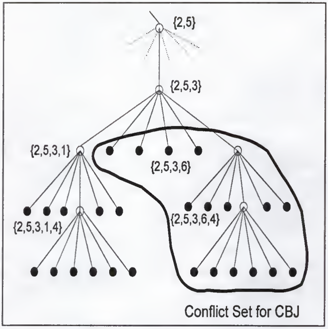


Figure 4.3 Partial Backtrack Tree with CBJ Conflict Set

4.4.2.1 Forward Checking (FC)

Forward Checking (Haralick & Elliot, 1980; McGregor, 1979) performs consistency checks from the current instantiation to future variables. The algorithm assigns a value to current variable from its domain then propagates the effect of that assignment to future variables by removing inconsistent values from their domains. Only when the future domain is annihilated (becomes empty), indicating that the current assignment has lead to a dead-end, are backtracking techniques employed. If a dead-end is reached the domains of the future variables are returned to their original state, and the next value is tried. If all values have been exhausted for the current variable domain, FC backtracks chronologically to the most recent successfully instantiated variable. This process continues until a complete solution is found or until all possible assignments have lead to a dead-end, in which case the problem has no solution. Forward Checking, in

contrast with backward checking algorithms, visits only consistent nodes, although not necessarily all of them.

4.4.2.2 Maintaining Arc-Consistency (MAC)

Similar to Forward Checking, Maintaining Arc-Consistency (Sabin & Freuder, 1994) focuses on checking future variables for arc-consistency. However, MAC not only checks the consistency of all potential future variables and deletes any values that are not supported by the current variable, it also checks for consistency between the newly identified future variables and their values. This type of incremental arc-consistency algorithm for re-establishing arc-consistency after each assignment reduces the size of the overall problem and thus has been shown to be efficient (Van Hentenryck, Deville and Teng, 1992).

4.4.3 Hybrid Backtracking/Forward Checking Algorithms

Various combinations of the previously described basic algorithms have been proposed to combine their advantages. For example Forward Checking and Conflict Directed Backjumping (FC-CBJ) tracks information about inconsistent variables and subsequently uses this information to determine the backtracking point. This algorithm has the advantage of establishing a conflict set to more efficiently direct the backward movement of the Forward Checking algorithm when it encounters a dead-end. Another extension, Backmarking (BM) improves the efficiency of the backtracking algorithms by adding a marking scheme (Gaschnig, 1977). Without the marking scheme consistency checks are performed to determine if the current instantiation of variables satisfies the constraint between the variables without regard to any historical checks that may have

already determined the consistency between these same two variables. The BM marking scheme reduces the number of consistency checks by employing the notion that if at the most recent node where a given instantiation was checked, the instantiation failed against some past instantiation that has not yet changed, then it will fail again. Therefore, all consistency checks involving it need not be investigated. It can also be assumed that a successful instantiation of some past instantiation that has not yet changed will succeed again. By marking the instantiations that have already been tested we avoid redundant consistency checks. The implication is that we need only check past instantiations that have changed or are "unmarked." Imposing a marking scheme on an algorithm does not change the nodes visited and therefore can extend any of the basic algorithms.

Kondrak and van Beek (1997) evaluated the efficiency of several backtracking algorithms with respect to the number of nodes visited and the number of consistency checks performed. They found that the hybrid backtracking algorithms such as Forward Checking and Conflict-Directed Backjumping, Backmarking with Backjumping and Backmarking with Conflict-Directed Backjumping tend to outperform the original algorithms. In fact, FC-CBJ has been shown to be among the best for solving hard problems (Kondrak & van Beek, 1997; Smith & Grant, 1995).

4.4.4 Improving Performance

The order in which variables are chosen for instantiation can play a significant roll in the performance of the algorithm. Variable ordering can be either static or dynamic. With a static variable ordering the order of the variables must be established prior to the constraint network being passed to the backtracking algorithm. A static order is in

contrast to a dynamic order of instantiation in which the decision of which variable to instantiate next is based on the current state of the search. Large portions of the search space can be pruned by employing the "fail-first principle" which chooses the most constrained variable first thereby forcing failures higher in the backtrack search tree (Van Hentenryck & Saraswat, 1996). A dynamic ordering algorithm that chooses the variable with the minimum remaining values (MRV) in its domain has been developed for use with both backtracking (Sabin & Frueder, 1994) and forward checking (Bacchua & van Ran, 1995) algorithms and has been shown to perform well on specific problems. The order in which the values are chosen will likewise determine how quickly the algorithm achieves a solution by allowing the most promising value to be assigned first. What constitutes "promising" is problem specific, for example if you are attempting to maximize profits, ordering the values for largest to smallest may achieve the best results.

4.5 Arc-Consistency Algorithms

Arc-Consistency algorithms complement, rather than substitute for, backtracking algorithms. Arc-consistency algorithms remove inconsistencies from the network generated by an instantiation that can never be part of a global solution. Removal of inconsistencies reduces thrashing (Mackworth, 1977). Mackworth (1992, p287) describes thrashing "as the repeated exploration of subtrees of the backtrack search tree that differ only in inessential features such as the assignment of variables irrelevant to the future of the subtree." By analyzing the various basis of thrashing behavior in backtracking, arc-consistency algorithms can eliminate the source.

Instantiating a variable impacts the domains of all prior variables and consistency algorithms must determine if the instantiation has violated any constraints or caused prior instantiations to be in violation. The most widely used consistency algorithms are AC-3 and AC-4. Unlike the previous algorithms that evaluated every arc, AC-3 rechecks consistency of only those arcs that could have been affected by current instantiation. AC-4 uses the same approach but maintains a special data structure that prevents repeated reexamination of pairs of values (Mackworth 1977).

Other algorithms exploit problem specific knowledge. For example AC-7 (Bessiere, Frueder & Regin, 1999) takes advantage of the bi-directional property of binary constraints to remove redundant checks. This algorithm works on the simple notion that a value *a* at node *I* (I,a) supports a value *b* at node *J* (J,b) if and only if (J,b) supports (I,a). Constraint bi-directionality properties allow the algorithm to perform fewer consistency checks and thus improve computational efficiency. Specifically, AC-7 can avoid checking if (J,b) supports (I,a) since it knows that the inverse, (I,a) supports (J,b), is true. Another arc-consistency algorithm, AC-8 (Chen, 1999) breaks the problem into smaller sub-problems then solves them sequentially.

4.6 Stochastic Search Algorithms

Stochastic search algorithms begin with a solution that may or may not be feasible and repairs it using a variety of techniques to achieve feasibility. This class of algorithms will normally achieve a feasible solution more rapidly than their systematic counterpart. The quality of the solution is predicated on the initial solution and the technique used for repairs. Stochastic algorithms have been proposed that use hill-climbing (Minton,

Johnston, Phillips & Laird, 1992), neural networks (Popescu, 1997; Kurgollus & Sankur, 1999), and genetic algorithms. Kanoh, Matsumote, Hasegawa, Kato & Nishihara (1997) suggest that genetic algorithms (GAs), due to their global search characteristics, provide effective solutions to CSPs that have many local optima. In fact, GAs have been used to effectively seed CSPs designed to solve ship maintenance scheduling (Deris, Omatu, Ohta, Kutar & Samat, 1997) and timetable planning problems (Deris, Omatu, Ohta, & Saad, 1999). Combining the two methods takes advantage of the strength of both constraint satisfaction methodologies and GA techniques. The genetic algorithm plays its role as a tool to generate promising solutions while constraint-based reasoning processes the constraints to ensure that the solutions are legal and valid. Kanoh et al. (1997) further modify the mutation process of the standard GA by substituting "viral infection" for standard mutation. A virus is defined as a partial solution to the CSP and is generated by the GA along with other candidate solutions. Crossover and infection then generate new candidate solutions. Infection gives direction to the evolution by substituting the genes of the virus for those of the individual generating a new candidate solution based on partial solutions proven to be consistent.

4.7 Constraint Programming

Constraint Programming finds variable instantiations that simultaneously satisfy all specified constraints while optimizing a stated objective. One strategy used for Constraint Programming is to model the problem as a CSP. After a feasible solution to the CSP has been found an additional constraint is added to represent the objective function. The new constraint requires that the objective strictly improve over the

objective value of the current CSP solution. This process is repeated until no feasible solution can be found. The last solution obtained prior to the problem becoming unsatisfiable is an optimal solution (Nonobe & Ibaraki, 1998; Potts & Smith, 1999).

4.8 Advertising Sales Application

Defining the television commercial time allocation problem as a constraint programming problem involves specifying the variables, domains and constraints as well as the ordering of variable instantiations and value assignments. Variables in the problem represent the airtime assignment to each bidder and the values assigned form a tuple indicating the shows that have been allocated. The allocation is subject to the following constraints:

- Maximize Seller Revenue: The overall objective function that must increase at each iteration.
- Reservation requirement: The aggregate sum of the all accepted bids must be greater than the sum of the discounted list prices for the commercials purchased.
- Maximum seller coverage: A seller specified maximum number of commercial slots must be sold in each show.
- Maximum spot availability: The number of commercial placements per pod cannot exceed the number of spots available to accommodate them. Assignments must be within the range of the domain.
- Buyer selection indicator: A buyer cannot be assigned units if his bid has been rejected.
- Campaign commercial length: The number of units assigned to a buyer in each show must be a multiple of 15 that corresponds with the campaign length. For example a buyer with a 30-second campaign must has either zero units in a show or a multiple of two.
- Anti-Clutter Restraints: No more than two 15-second commercials can appear in the same pod.
- Maximum Commercials per Show: The number of correct length commercials in each show must not exceed the bidder specified maximum. This could be as small as zero which effectively eliminates that show from consideration.

- Demographic gross impression guarantee: The demographic gross impressions summed over all selected shows and equivalized by commercial length must meet or exceed the required reach for the specified demographic group for each bidder.
- Show placement minimum: Winning bidders should be placed in the shows they requested. The minimum number of requested show assignments should correspond with the lower bound specified by the bidder.
- Show placement maximum: Winning bidders should not be placed in an identified show more than the maximum number of times indicated by the upper bound specified by the bidder
- Pod protection: No two buyers advertising the same category product can be placed in the same pod if at least one has a 30-second commercial.
- Bid amount not exceeded: The sum of the discounted list prices for the allocation of units to each bidder must be less than or equal to their amount bid.

The constraint programming methodologies will be employed to determine variable instantiation and manage the large domains of the problem. Variable instantiation ordering involves sorting the bidders by some criteria of interest. Both variable instantiation and value ordering will be discussed in detail in subsequent chapters.

4.9 Summary

This chapter presented an overview of constraint satisfaction problems and constraint programming. The former involves finding a set of values that simultaneously satisfies all constraints while the later extends the feasible solution to the CSP by including an objective function that is iteratively tightened to find an optimal solution. We looked a various algorithms designed to discover satisfying allocations. Arc consistency techniques, algorithms employed to control the consistency of domains were reviewed. They are important to our research as they provide an efficient means of managing the large domains of our problem. Finally, we introduced our auction mechanism in constraint programming language.

CHAPTER 5
HUERISTIC DEVELOPMENT

5.1 Introduction

A direct attack on solving problem P1 is probably doomed. For example, in a representative problem with 325 bidders competing for 587 units in 109 pods across 24 shows P1 generates approximately 278,000 binary variables and 587,000 constraints. A heuristic is clearly needed. The heuristic used to solve our combinatorial auction in real time is developed in this chapter. The overall approach is outlined in Section 5.1. It incorporates a mixture of problem aggregation with linear, constraint and dynamic programming methods. Once we give the overall approach, we focus on the particulars. Many of the decisions necessary to discover an optimal allocation of goods can be determined at an aggregate, or show level rather than at the unit or pod level. Working with the aggregate problem dramatically reduces the size of the problem. Descriptions of the aggregate sub-problems are presented in section 5.2. The results of these sub-problems are incorporated into the master problem in section 5.4. The aggregate problems use constraint programming methods as an efficient way to collapse the size of the original search space. Section 5.3 defines the constraint programming aspect of the heuristic that will manage the domains from which the allocations are chosen. Constraint programming employs simple computer programming logic to replace complicated

equations thus provides a more efficient method of ensuring that constraints are enforced. Finally, branch and bound methodology is used to search for an optimal allocation, search guiding heuristics and fathoming criteria are presented in section 5.5.

5.2 Overview

The solution methodology employed to allocate units in our Incompletely Specified Combinatorial Auction is fairly complicated incorporating several techniques. Figure 5.1 presents an overview of the procedures.

The auction begins with the collection of bids. Each bid is subjected to an initial feasibility check to ensure that it meets minimum requirements for entry. This is accomplished by solving an aggregate problem defined below. Once all bids have been tendered, an initial feasible solution is generated with the use of heuristics that will be defined in detail in section 5.4. An upper-bound is established using a linear relaxation of a second type of aggregate integer program. This bound is used to judge the quality of our solutions. The best solution to date is then used to start a branch and bound search. At the end of each auction round, the selected stopping criterion is checked. If stopping conditions are not met, bidders are informed of the results. Loosing bidders will have the opportunity to change their bids commensurate with their behavior profile. When all stopping conditions have been met, the current bid amounts are replaced with bidder reservation prices (but otherwise unaltered) and are used to compute a solution to be used for the efficiency calculation.

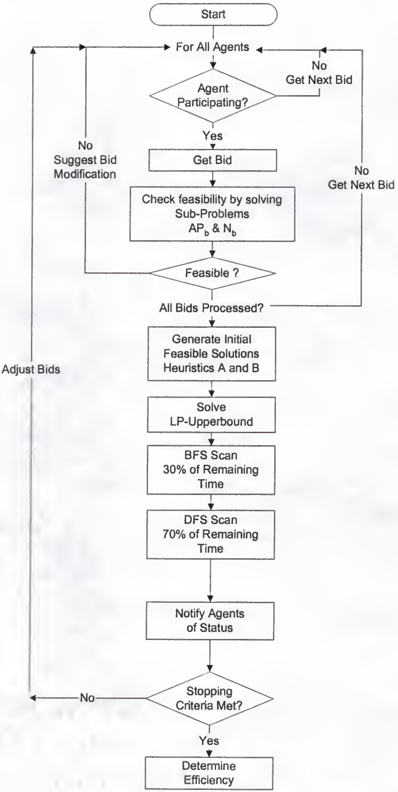


Figure 5.1 Auction Overview Auction Flowchart

5.3 Aggregate Sub-Problems

The majority of constraints involved in the auction problem can be determined by examining the allocations of each show rather than a pod or unit level allocation. The overall driving heuristic is a greedy allocation of show slots to bidders. Each bid is considered sequentially, conditional on the tentative allocations made to other bidders. By aggregating to the show level we reduce the size of the problem and thus enhance our ability to achieve a solution. Let $\delta(x)$ be the normal Kronecker delta, i.e.,

$$\delta(x_{s,b}) = \begin{cases} 1 & x_{s,b} > 0 \\ 0 & x_{s,b} = 0 \end{cases}$$

Also, let $x_{s,b}$ be the number of allowable units that bidder *b* may purchase in show *s* and $X_{s,b}$ be the current domain of $x_{s,b}$. The domain of $x_{s,b}$ will change as other variables associated with earlier accepted bids are instantiated either because units become unavailable or some constraint such as pod protection or maximum spots per show would be violated. As we show in Section 5.4, $X_{s,b}$ incorporates all the constraints given in Equations (1.2)-(1.11d) except for demographic reach and the desired shows constraints. These latter two are handled directly in the following aggregate problem.

We define the aggregated problem, (AP_b) for each bidder as

$$(AP_b) \quad \sigma_b = (1-r) \min_{x_{s,b} \in X_{s,b}} \sum_{s=1}^{S} L_s x_{s,b}$$

$$\sum_{s=1}^{S} D_s x_{s,b} \geq T_b$$

$$\underline{H}_b \leq \sum_{s=1}^{S} h_s \delta(x_{s,b}) \leq \overline{H}_b$$

When there are no current other assignments, the objective value is labeled σ_b which gives the minimum discounted show costs needed to satisfy all problem constraints (1.2) – (1.11d). When there are current assignments meeting (1.2) – (1.11d), then (AP_b) provides an assignment for this bid (if it has a feasible solution) that, together with the current assignments, meet all constraints (1.2) – (1.11d). By minimizing the discounted costs, we hope to also, in total, satisfy the one remaining constraint, constraint (1.1), the seller reservation price constraint.

If there is no feasible solution, then set the $x_{u,p,s,b}$ variables to zero. Otherwise, a solution to (AP_b) can be expanded to yield $x_{u,p,s,b}$ values by recovering a combination yielding the correct entry in $X_{s,b}$. There may be many such combinations. No currently protected pod will be violated by these $x_{u,p,s,b}$. If (AP_b) has a feasible solution, then we set $y_b = 1$. If not, we set $y_b = 0$. The resulting y vector indicates the eligible participants for this round. The remaining decision variables in the original problem (P1) can be recovered by analysis of the expanded solution $x_{u,p,s,b}$'s. A simple count of the ultimate allocation of *x*'s for each bidder *b* in each show *s* and pod *p* will determine the value to assign $f_{p,s,b}$. If the sum of the *x*'s is greater than 1 in a pod then that bidder has more than a 15-second commercial in that pod and $f_{p,s,b}$ is set to 1. This same number when compared to the set of allowable commercial lengths, $N_{s,b}$, yields the values to assign the $I_{p,s,b,i}$ variable. If a commercial of length *i* appears in the final allocation in a particular pod *p* of show *s* for this buyer $I_{p,s,b,i}$ is assigned a value of 1 otherwise it is set

to 0. This same counting technique aggregated to the show level will identify the appropriate $j_{s,b}$'s to set to1 indicating that buyer b has a presence in show s.

The aggregate problem (AP_b) is solved using dynamic programming. Notice that, with the exception of the last constraint, this is a straightforward Knapsack program. The final constraint can make this a non-linear problem (because of the Kronecker operation) if either or both $\underline{H}_b$ and $\overline{H}_b$ are greater than zero and the h's have values necessitating the consideration of these constraints. We utilize one of several dynamic programming routines designed to solve the sub-problem, the choice of which depends on the values of $\underline{H}_b$ and $\overline{H}_b$ and the nature of the h's that have been selected. The dynamic programs provide exact optimal solutions to (AP_b). However, these can take some time to solve since the T_b values may be large. At various points, to be discussed, we use a heuristic based on linear programming ideas to give good (often optimal) solutions to the aggregate problem (AP_b). We call these methods, FastAP.

Just as we utilize one of several dynamic programming routines designed to solve the sub-problem, the choice of which depends on the values of $\underline{H}_b$ and $\overline{H}_b$ and the nature of the h's that have been selected, we also have different FastAP approaches. Overall, however, they are based on linear programming relaxations. When the show selection constraints aren't needed, a straightforward LP Knapsack problem is solved. Otherwise, a slightly more complicated version is employed to satisfy the show selection constraints. Each solution is refined using problem reduction methods which shrink the domains based on simple dominance tests.

A problem similar to (AP_b) is given below and proves useful in several situations. When no assignments have been made, let

$$(MN_b)\quad \eta_b = \min_{x_{s,b} \in X_{s,b}} \sum_{s=1}^{S} x_{s,b}$$

$$\sum_{s=1}^{S} D_s x_{s,b} \geq T_b$$

$$\underline{H}_b \leq \sum_{s=1}^{S} h_s \delta(x_{s,b}) \leq \overline{H}_b$$

Using the same techniques deployed to solve (AP_b) we are able to determine the minimum number of units, η_b, needed to satisfy all of the constraints (1.2) – (1.11d) for bid b.

5.4 Domain Management – Constraint Programming

Effectively managing the $x_{s,b}$ domains is extremely important to the heuristics ability to reach a timely solution to this problem. Due to its combinatorial nature and the large number of available units, the problem size can quickly become insurmountable. We employ constraint programming concepts as a means of coping with these sizes. Simple programming logic can replace complicated logic equations providing a more efficient method of ensuring that constraints are enforced. By dynamically reducing the size of the domain as units become unavailable and only generating the combinations that satisfy an individual buyer's constraints we avoid total enumeration. We utilize a "greedy" assignment methodology were we assign one bidder at a time and then adjust the remaining domain to reflect the resulting slot assignments.

In the previous section we developed an aggregate problem used as a basic component for solving P1. This formulation relies heavily on the domains $X_{s,b}$. The following procedure is used to determine the domain for each bidder given the current available slots and previous allocation of bid.

Step 1. Let $X_{s,b} = \phi$ and $\gamma(N_{s,b}) = \begin{cases} 0 & 1 \notin N_{s,b} \\ 1 & 1 \in N_{s,b} \wedge 2 \notin N_{s,b} \end{cases}$

$\gamma(N_{s,b})$ is set to 0 if there are no 15-second commercials in buyer b's campaign indicating that the anti-clutter constraint is not applicable to this campaign. If the buyer is running 15-second commercials and not 30-second spots, this parameter will be set to 1. When $\gamma(N_{s,b}) = 1$ special consideration must be given to ensure that no more than 2 15-second commercials for this bidder appear in the same pod. Since the buyer is not running 30's two units in a pod must be individual 15-second commercials.

Step 2. Let $g_{p,s}$ be the remaining number of open slots in pod p of show s. Open slots are defined as those that are not yet owned. Additionally, if a competitor owns 2 or more slots in a pod p of show s then no units in that pod are deemed available. If a competitor owns a 15-second slot in that pod then the pod is "weekly owned" and only 1 unit is considered open. This methodology ensures pod-protection and gives priority to current owners of weakly owned pods.

Step 3. For each pod define $X_{p,s,b} \equiv \{n \in N_{s,b} : g_{s,b} \geq n\}$. These are the campaign lengths feasible for each pod. The lengths allowed in each pod are entirely dependent on the number of units available and the allowed campaign lengths. For example if

there are 3 units available in a pod and a bidder is running 30- and 60-second commercials they could only have a 30-second (2 unit) spot in that pod.

Step 4. Let Θ_i be the set of all combinations of size $i = 1,\ldots,K_{s,b}$ of the sets $X_{p,s,b}$ (used no more than once in a combination when $\gamma(N_{s,b}) = 0$ or no more than once with the exception that two 15-second units are allowed in the same pod if $\gamma(N_{s,b}) = 1$. The aggregate over each show is $X_{s,b} = \bigcup_{i=1,\ldots n} \Theta_i$. This enforces the anti-clutter constraint.

Step 5. Remove each $x_{s,b} \in X_{s,b}$ from $X_{s,b}$ where the number of currently committed slots plus $x_{s,b}$ exceeds C_s, thus limiting the number of assignments in each show to no more than the maximum allowable.

An example will help clarify how the domains are computed. Assume for some show *s* and buyer *b* we have the following:

- $C_s = 24$, and the current number of committed slots for that show is 17, leaving seven open units.
- $N_{s,b} = \{1,4\}$ or the campaign consisting of only 15- and 60-second commercials.
- $K_{s,b} = 4$ (any combination of the allowable 15- or 60 second spots totaling at most 4 correct length commercials are allowed in show *s*).
- P = 4. There are 4 pods in show *s*
- The number of open slots in each of the four pods are as indicated. $g_{1,s} = 5$, $g_{2,s} = 0$, $g_{3,s} = 7$, $g_{4,s} = 3$

The above states that this bidder is running campaigns of length 1 and 4 ($N_{s,b} = \{1,4\}$), can have at most 4 commercials in this show ($K_{s,b} = 4$), that there are 4 pods in the show (P = 4) having 5, 0, 7 and 3 remaining slots available to this bidder. The zero availability in pod two may have resulted from pod protection given to another bidder in a prior step

of the solution methodology or it may simply have been completely used by prior assignments.

Step 1 gives $X_{s,b} = \phi$ and $\gamma(N_{s,b}) = 1$.

Step 2 is specified above by the *g* values.

Step 3 gives $X_{1,s,b} \equiv \{1,4\}$, $X_{2,s,b} \equiv \phi$, $X_{3,s,b} \equiv \{1,4\}$, $X_{4,s,b} \equiv \{1\}$.

Step 4 gives the following. The examples illustrate possible assignments.

$\Theta_1 = \{1,4\}$ (different combinations consisting of 1 correct length spot)
$\Theta_2 = \{2,5,8\}$ (e.g., 5 = a length 1 in pod 1 and length 4 in pod 3)
$\Theta_3 = \{3,6,9\}$ (e.g., 6 = a length 1 in pods 1 and 4 and length 4 in pod 3)
$\Theta_4 = \{4,7,10\}$ (e.g., 7 = two length 1's in pod 1, a 4 in pod 3, and a 1 in pod 4)

Then $X_{s,b} = \{1,2,3,4,5,6,7,8,9,10\}$.

Step 5 requires us to remove 8, 9 and 10 since we have at most 7 units available to assign. Thus the final $X_{s,b} = \{1,2,3,4,5,6,7\}$ is the domain for show s from which we select allocations to satisfy buyer b's requirements. Although this example might suggest the contrary, a domain need not have all the integers between the upper and lower element.

5.5 Master Problem

An overview of the master problem is presented in Section 5.2, Figure 5.1. The goal is to find a solution that maximizes seller profits as specified in problem P1 of Chapter 3 while satisfying constraints (1.1) to (1.11d). The approach employed utilizes the heuristics described in previous sections and methods defined here to determine an allocation that approaches optimality.

To establish a good initial solution to the allocation problem, consider the following two greedy algorithms. Assume we are given B bids. The two algorithms differ only by the sort criteria used in step 1.

Step 1: Repeat the following until all bids have been processed

Sort the remaining bids by some criterion of interest with the most desirable bid designated as the top-most bid. See sorting criteria 1 and 2 below.

Solve the aggregate sub-problem for the top-most remaining bid and make the appropriate assignments to the variables of P1.

Step 2: While the amount bid by the selected bidders is less that the seller's reservation price for the collection of allocated units, i.e.

$$\sum_{b=1}^{B} a_b y_b < (1-r)\left(\sum_{b=1}^{B}\left(\sum_{s=1}^{S}\left(\sum_{p=1}^{P_s}\sum_{u=1}^{U_{p,s}} x_{u,p,s,b}\right)L_s\right)\right)$$

Sort the remaining feasible bids by some criterion of interest with the least desirable bid designated as the top-most bid. See sorting criteria 3 below.

Set the top-most active bid's aggregate sub-problem solution to infeasible and remove any current allocations to this bidder.

Furthermore, the above procedure yields a feasible solution to the auction problem as is now shown. First, Step 2 guarantees the feasibility of the reservation requirement (1.1). The construction of the domains for each aggregate sub-problem plus the constraints of (AP_b) assures the feasibility of all the remaining constraints. See Figure 5.2 for an overview of this heuristic.

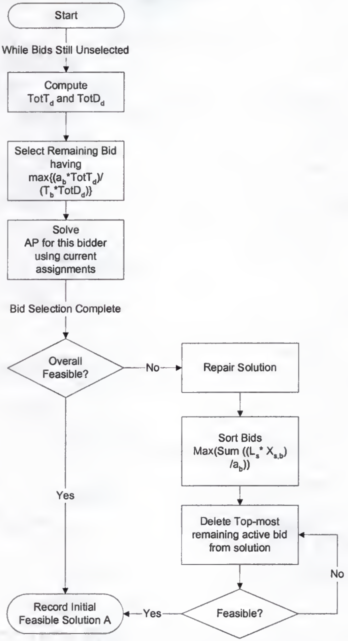


Figure 5.2 Heuristic Flowchart

5.5.1 Sorting Criteria 1

The first sorting criteria is designed to order the bidders in such a manner that those that contribute the most to maximizing seller revenue are assigned first. To accomplish this we find the bidder that solves the following

$$\max\left(\frac{a_b}{(T_b)_d}\frac{TotT_d}{TotD_d}\right),$$

where

$$TotT = \sum \text{remaining demographics} * \mathrm{L_s} \text{ and}$$
$$TotD = \sum \text{remaining required demographics } * \mathrm{a}_b,$$

and d is the demographic required by bidder b. We assume, as is industry practice, that each bidder's demographic requirement T_b is in only one demographic category. Simply stated, the bidder with the highest bid per thousand demographic requirements normalized by the cost of the specific demographic category and demand within that category is selected.

5.5.2 Sorting Criteria 2

The ratio of the actual bid amount, a_b, and σ_b, the minimum possible cost allocation for that bid b, defines sorting criteria 2. The sorting equation is as follows

$$\max\left(\frac{a_b}{\sigma_b}\right)$$

5.5.3 Sorting Criteria 3

The following sorting criteria will force those bidders with the largest actual cost to bid ratio to be removed first, enhancing the auction's ability to achieve a feasible solution.

$$\max\left(\frac{\sum_{s=1}^{S} x_{s,b} L_s}{a_b}\right)$$

5.6 Branch and Bound

Branch and bound techniques are employed to investigate the various combinations of bids that will maximize seller revenue. Total enumeration of the various combinations is impossible in any reasonable amount of time, thus we utilize heuristics to guide our branching behavior. At each branch, we take the partial solution from predecessor branches and solve (AP_b) (or FastAP).

The amount of time allotted to computation in each round is pre-defined. Therefore we use time remaining as a guide to the search process. After preprocessing is complete, an initial solution determined and an upper bound on P1 computed, the remaining time is used as follows. Thirty percent is spent in a Breadth First Search (BFS), the rest is dedicated to a Depth First Search (DFS).

5.6.1 Breadth First Search (BFS)

The Breadth First Search extends to three levels. This means that it looks at all orderings of all combinations of 3 bidders – time permitting. Below level three, depth first search is used but is limited to a relatively small number of branchings (we use five times the number of bidders). Bids are initially ordered by the heuristics previously described so as to rank them such that the top-most bids contribute the most to maximizing revenue. However, conflicts between these bids may prevent all of these most desirable bids from achieving an allocation. The order in which the bids are processed affects the allocation so all permutations of the three bids are explored. During the Breadth First phase, the FastAP heuristic is used.

For example, the BFS systematically explores all permutations of ordered bids {1,2,3} then bids {1,2,4}, {1,3,4}, {2,3,4}, etc, expanding each of the permutations with a DFS. This process continues until 30% of the remaining computational time has been exhausted at which point, the final 70% of computing time is dedicated to a strictly Depth First Search.

5.6.2 Depth First Search (DFS)

A Depth First Search is employed during the final 70% of computational time to seek out the best combination of bids. This search is conducted in two stages, the first solves (AP_b) exactly using dynamic programming and lasts for 60% if the time allotted. Stage 2 utilizes FastAp and runs until the conclusion of the computational time.

5.6.3 Fathoming Criteria

Some fathoming criteria that are used to limit the branch and bound search follow.

1. Based on reservation price.

   Recall that σ_b is the objective value of a solution to the aggregate problem (AP_b) where no prior assignments have been made. Then any branching exploration should be restricted to cases where the amount bid by the feasible bidders meet the minimum cost allocation that satisfies all constraints. In other words, don't consider any selection of bids where

$$\sum_{b=1}^{B} a_b y_b < \sum_{b=1}^{B} \sigma_b y_b$$

2. Based on best feasible solution value to date.

Don't explore any assignment of y's giving an objective value to P1 that is less than the current best feasible solution subject to the amount of inventory available. That is, don't explore any y's satisfying:

$$\sum_{b=1}^{B} a_b y_b < \text{Best Feasible Solution Value,}$$

with

$$\sum_{b=1}^{B} \eta_b y_b \le \sum_{s=1}^{S} C_s$$

5.7 Determining an Upper Bound to P1:

The overall problem (P1) is upper-bounded by:

$$U = \max_{y \in Y} \sum_{b=1}^{B} a_b y_b$$

$$\sum_{b=1}^{B} (\sigma_b - a_b) y_b \le 0$$

$$\sum_{b=1}^{B} \eta_b y_b \le \sum_{s=1}^{S} C_s$$

$$0 \le y_b \le 1$$

This formulation incorporates the results of the aggregate problem (AP_b) and the minimum number of units problem (MN_b) defined earlier together with constraint (1.1). The bound provides a way to judge the current best solution to all constraints (1.1) - (1.11d). This problem is simple to solve since a bounded-variable Simplex method with only two constraints is trivial.

5.8 Summary

To cope with the size and complexity of the problem and facilitate reaching a solution in real time a heuristic was developed and described in this chapter. The

heuristic solution to the combinatorial optimization problem (P1) is by no means trivial. Various techniques were employed to guide our quest for an optimal allocation, including a mixture of problem aggregation with linear, constraint and dynamic programming methods as well as branch and bound search. Subsequent chapters will test the efficacy of the methods engaged.

CHAPTER 6
SIMULATED BIDDING AGENT DEVELOPMENT

6.1 Introduction

Our experiment consists of simulating the execution of our auction under various conditions to analyze the mechanism's performance. To facilitate this we generate players that reflect the characteristics of the real world environment. An analysis of data received from a representative of a major television network provides a statistical basis for player typing. Each player or agent represents an individual bidder with defined parameters that reflect media buying practices within the industry. The parameters include bidder product requirements. To establish product requirements necessitates defining the desired demographic category and gross rating points (GRP) required as well as bidder reservation prices. Show selection for each agent includes a list of desired shows and an upper and lower bound to the number required. The number of commercials allowed in each show and the type of product being sold will also be specified. Finally, a bidding strategy is defined for each agent type that governs the agent's behavior during the auction's execution. A visual summary of the entire process of generating a bid agent is presented in Figure 6.1. We describe each process in detail in the remaining sections of this chapter. Every attempt was made to depict as many types of bidders as necessary to accurately represent the behavior of the market participants.

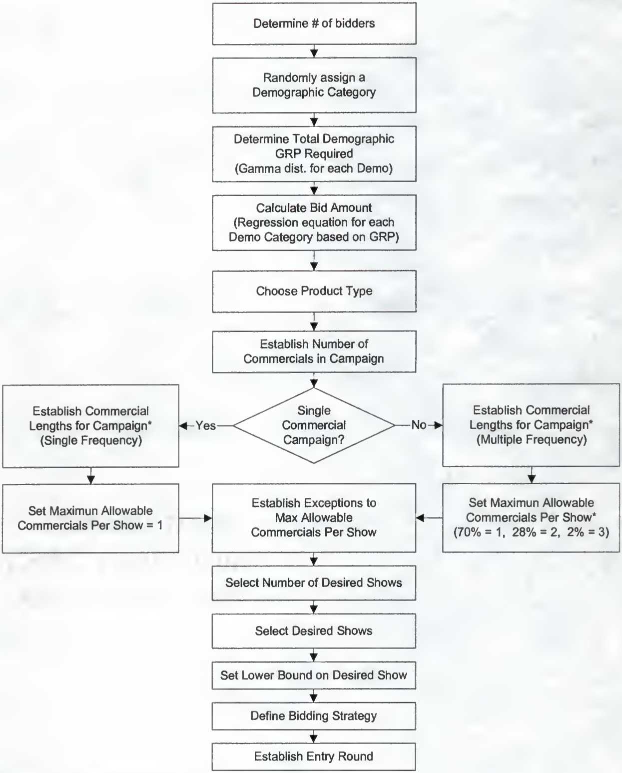


Figure 6.1 Flowchart of Initial Agent Generation

Classical auction theory assumes homogeneity among auction participants, however new evidence suggests that in electronic auction several types exist. Bapna, Goes and Gupta (1998) describe three distinct bidder categories, Evaluators, Participators and Opportunists, these will be used as a basis for our bidder definitions.

6.2 Data Analysis

The data analyzed for this research was provided by one of the major television networks. The source prefers to remain anonymous. Within this industry information, especially pricing and inventory availability, is extremely proprietary and jealously protected, therefore we have attempted to disguise the results of our analysis while at the same time benefit from the discovery of patterns. Our analysis consisted of reviewing two representative weeks of actual airtime allocations. The data consists of 150 unique bidders representing 209 purchases to acquire 1290 units of airtime across 48 shows. Not all information needed to formulate our buyer typing was known either because of our inability to access the appropriate data or the lack of representative data. We were required in some instances to make assumptions. Some of our assumptions stem from anecdotal evidence provided by our information source while others are based on expectations of a rational buyer in a competitive market. During the following discussion of buyer typing we will identify and justify the assumptions. The patterns and frequencies identified will be used as a basis from which we randomly generate characteristics of each agent and thus may not be exactly duplicated in our experiments.

6.3 Number of Buyers

The number of bidding agents to generate was our first consideration. The two weeks of data consisted of roughly 90 to 120 buyers who were awarded "upfront" allocations in each week. Our industry source indicated that 300 to 350 buyers participate in "upfront" with their allocations distributed across the weeks of the year. As a conservative estimate we generate 325 bidding agents.

6.4 Demographic Category and Total Gross Rating Points

Media buyers must achieve a certain amount of demographic exposure or gross rating points within a specific demographic group to satisfy their campaign needs. The A. C. Neilson Ratings for network television is the measure used to determine the number of people exposed to a particular program and hence to the commercials appearing in that show. The ratings are broken down in various categories representing the gender and age of the viewing audience. Six of these categories are typically associated with primetime advertising sales; Women, Men and Adults within age groups 18-49 and 25-54. Category assignments for each agent were randomly chosen from a uniform distribution over the values 1 to 6. An analysis of the data revealed that the amount of demographic gross rating points required by each buyer followed a gamma distribution. Figure 6.2 shows the distribution of the buyer frequencies over the total Gross Rating Points (GRP) averaged across all categories. The individual category distributions appear in Appendix A. Appropriate MLE estimates of Gamma distribution parameters $\hat{\alpha}$ and $\hat{\beta}$'s were generated for each category. Simultaneously satisfying the following two equations by

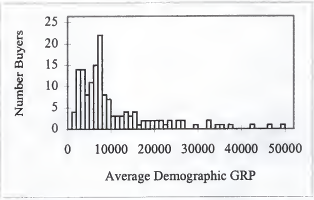


Figure 6.2 Average Gross Rating Points

varying $\hat{\alpha}$ gives us the estimated parameters.

$$\ln \hat{\beta} + \psi(\hat{\alpha}) = \frac{\sum_{i=1}^{n} \ln X_i}{n},$$

and

$$\hat{\alpha}\hat{\beta} = \overline{X}(n),$$

where $\psi(\hat{\alpha}) = \Gamma'(\hat{\alpha}) / \Gamma(\hat{\alpha})$ is the digamma function with Γ' denoting the derivative of Γ (Law & Kelton, 1991). The $\hat{\beta}$ values were then varied to maximize the goodness of fit measure. The Kolmogorov-Smirnov one-sample test was used to determine the goodness of fit between the estimated Gamma distribution and the sample values. The cumulative distribution for the theoretical Gamma distribution is compared to the cumulative distribution of the actual data. The maximum deviation from the theoretical distribution must be less than or equal to a critical value defined for the test. Results of the test are presented in Table 6.1 and validate at a 0.05 significance level that the sample came from a population having a Gamma distribution (Siegel, 1956). Using Phillips

(1971) gamma variate generator and the estimated $\hat{\alpha}$ and $\hat{\beta}$ for the appropriate category we are able to generate a representative random demographic value to assign an agent.

Table 6.1 Goodness of Fit Test

Kolmogorov-Smirnov One-Sample Test		
Significance Level = .05 (2-tailed)		
Critical Value = .09407		
Max Difference		<= Critical Value?
Demo 1	0.0675	Yes
Demo 2	0.0657	Yes
Demo 3	0.0777	Yes
Demo 4	0.0721	Yes
Demo 5	0.0696	Yes
Demo 6	0.0751	Yes

6.5 Bidder Reservation Price

The data analyzed for our study includes the amount that each successful buyer paid for each of their allocated commercials with the associated placement information. We can safely assume that the buyer's reservation price over all their units is at least the sum of the individual unit values. We also know the network's estimated gross rating points for the shows within the sampled weeks. The seller's rating estimates are not common knowledge among the buyers, instead buyers base their demographic requirement calculations from approximations discovered from historical ratings of prior seasons and market research on new programming. These approximations are fairly accurate in reflecting the seller's figures, therefore we apply the known seller rating estimates to determine roughly how many demographic gross rating points the buyers in the data desired by summing the demographics over the shows they were allocated. We

then equivilize the demographics to reflect the length of commercials. For example, a 30-second commercial would receive twice the demographic exposure of a 15-second spot.

A strong linear relationship was found to exist between the price paid for the units and the equivilized demographic GRP for the associated shows but showed signs of heteroscedasticity. A natural logarithm transformation of both variables remedied the increasing variance. See Table 6.2 for a summary of the fit.

Table 6.2 Regression Fitness Statistics

LNPrice = Constant + (Coeficient * LNDemo)				
Demo	**R Square**	**F**	**t**	**Sig.**
1	.836	1056.437	32.503	.000
2	.837	1065.432	32.641	.000
3	.835	1047.752	32.369	.000
4	.834	1038.267	32.222	.000
5	.844	1118.842	33.449	.000
6	.846	1135.407	33.696	.000

By regressing logarithmic price against the logarithmic equivilized total demographic GRP within each demographic category we were able to establish equations for determining a representative reservation price, see Appendix B for the detailed results. Each category's regression equation was determined from 209 observations. Based on the demographic category and the total demographic GRP's formulated in the previous steps we can calculate, from the appropriate regression equation, individual reservation prices to assign the agents that reflect the amount and type of product desired. For the purposes of this study we assume buyers' valuations can be represented by a step function

$$v_b = \begin{cases} 0 & \text{if constraints are not met,} \\ \text{Reservation Price} & \text{if constraints are met} \end{cases}$$

An allocation has no value to the buyer unless all constraints have been met, while any allocation that satisfies the constraints is valued at his reservation price. Buyers are trying to satisfy explicit campaign goals at a minimal cost thus adding additional units to the minimal constraint satisfying allocation does not add value.

The regression gives us the reservation prices representing those buyers who were successful in achieving an allocation. We recognize that there may have been participating buyers that were not successful in securing an allocation and others that were not forced to pay their true valuations, therefore we increase the variance of the actual reservation price assigned by a random amount uniformly distributed between 20% above or below the calculated figure.

6.6 Commercial Lengths and Frequency

An analysis of the historical data provided indicates that approximately 76% of bidders aired multiple commercials within the campaign week, while 24% placed only one spot. Our agent demands reflect these statistics. Within the two groups, multiple or single placement, we were also able to determine a frequency of the lengths of commercials aired. Table 6.3 details the breakdown upon which we programmed our agents. Within the multiple commercial campaigns there were instances of mixed commercial lengths where a single buyer uses a variety of commercial lengths within the same campaign that we reflect in defining our agents.

Table 6.3 Number of Commercials in Campaign

% / Length	Single Unit	Multiple Units
15's	16%	21%
30's	76%	48%
60's	8%	2%
15's & 30's		24%
30's & 60's		2%
Other mixed		3%

6.7 Limits on Number of Commercials

Dispersing commercials across various shows ensures that as many different viewers as possible are exposed to an advertiser's message. Generally, advertisers limit to one the number of commercials in each show. The data confirmed that 70% of advertisers with multiple commercials in the campaign week allowed only one spot per show, while 28% accepted up to 2 and 2% permitted as many as 3. We capture these same frequencies within our agents. Exceptions to the norm involved either forbidding placement entirely or increasing the number of spots allowed in a particular show by one or more units. We pattern the exceptions in our agents as described in Figure 6.3. We have allowed 30% of all agents to have no modification to the number of commercials per show. Another 62% will forbid placement in some portion of the shows that they have not specifically requested, while 8% permit more than the normal maximum number of spots per show.

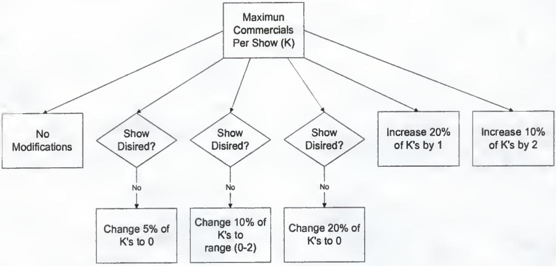


Figure 6.3 Maximum commercials per show modification

6.8 Selection of Product Type

The type of product advertised by our participants impacts on the allowable placement of the commercials. The industry practice of not allowing two commercials promoting the same type of product to appear in the same commercial break if one is at least 30-seconds long, referred to as "pod protection," makes distribution of product types across buyers an important consideration in our agent development. We were able to ascertain from the data provided the frequency of the various product types and use those frequencies to establish a representative mixture in our agents. We discovered a concentration in only a few product types such as retail, automobile sales, and restaurants.

6.9 Selection of Desired Shows

The number and identification of desired shows assigned to each agent were determined randomly. The number of shows was based on the arbitrary distribution listed in Table 6.4. Once we had assigned the number desired, the selection of specific

shows was determined from one of three show orderings, minimum cost per thousand (CPM), maximum demographics in that agent's demographic category and minimum anticipated cost. Our assumption is that a rational buyer would want a selection of shows

Table 6.4 Number of Shows Desired

Percent of Bidders	Range
30%	No Preference
30%	1-3 Shows
30%	4-10 Shows
10%	11-15 Shows

that will best satisfy their demographic needs within a particular category. Choosing from shows with the minimum CPM in that demographic category ensures that the buyer is getting the most cost efficient placements. Although the buyer does not know with certainty the CPM for each show we assume she has value and demographic estimates that are representative of those held by the seller. Therefore we sort the shows using the data on list prices and demographic GRP's provided by our industry expert. We further assume that selecting desired shows based on the minimum CPM to be the overwhelming choice among the buyers and assign shows to 60% of our agents using this ordering. Another approach is to choose from the shows with the highest demographic gross rating points regardless of the price. 30% of our agents employ this strategy that places the buyer in shows promising the most exposure to the viewers in her desired demographic category. To achieve the greatest frequency of placement buyers may want only the least expensive shows, 10% of our agents are assigned minimum cost shows. Table 6.5 indicates the allotment across the agents for each ordering.

Table 6.5 Show Selection Criteria

Percent of Bidders	Order
60%	Min CPM
30%	Max Demo
10%	Min Cost

Establishing the lower bound of the number of selected shows required by each bidders is determined as follows: 40% of agents place no bounds on the number of shows required, another 40 % set their lower bound at 2-3% of the selected shows. The remaining modifications are more restrictive with 10% requiring 4-6% of the selected shows, 6% require 6-8% and 4% having tight bounds, see Table 6.6.

Table 6.6 Desired Show Bounds

Bounds	% of Bidders
No Bounds	40%
2-3% of # Selected	40%
4-6% of # Selected	10%
6-8% of # Selected	6%
Tight	4%

Upper bounds were set at the number of shows selected. The frequencies were chosen at random but reflect anecdotal evidence. Buyers are described as routinely indicating that they would prefer as many of a particular type of programming as possible, but generally don't specify the exact the number required. Examples could be a desire by the buyer to appear mostly in situation comedy shows, or possibly only in news programming. Rarely, an advertiser may demand to appear in a specific show or group of

shows and if the network is unable to grant their request will not accept alternative placement. This last example represents a bidder with tight bounds.

6.10 Bidding Strategy

A buyer's bidding strategy dictates their behavior during the execution of the auction. We model this behavior by how bid modifications are made throughout the course of the auction and the decision on when the bidder places her entering bid. Recent research on electronic auctions, such as the Multiple Vickery Auction and the English Auction for multiple items, has identified three specific types of bidders; Participants, Evaluators and Opportunists (Bapna et al., 1998). We use these types as a foundation on which to model our agents' bidding strategies. Evaluators have a clear idea of their valuation and place one bid early in the auction that reflects that value. If outbid they do not reenter the auction. Participators take a dynamic role by bidding early and actively modifying their bids as necessary to remain active during the course of the auction. Opportunists, or those bidders who place a minimum bid just before the auction closes, are not explicitly modeled as agents in our experiments because the end of the auction in not generally known. The Incompletely Specified Combinatorial Auction does allow bidders to enter in late rounds but since our auction is semi-sealed with soft closing rules the required signal information to assist the Opportunist in determining the minimum bid and or closing round is not available. Thus there is little incentive to delay bidding and we assume that the majority of bidders will enter in the early rounds. See Table 6.7 for a breakdown of the initial entering round parameters applied to the bidding agents.

Table 6.7 Initial Entry Round

EntryRound	Percent
1	80%
2	10%
3	2%
4	2%
5	1%
7	1%
10	1%
12	1%
15	2%

The two major approaches to bid modification involve changing the bid amount or loosening constraints imposed on the choice of shows to fulfill the demographic GRP requirements. Bid modification tactics are broken into six styles, 5 represent the Participator type and the other takes on the characteristics of an Evaluator. We further divide Participators into five subcategories depending on how they make their bid modifications. The multidimensional nature of the ISCA bid allows for a variety of modifications. Of the Participators, two modify their bid amount only, two modify both their bid amount and the constraints imposed on commercial placement and one modifies only the constraints. We define BidAdjustors as those bidders who modify the price they are willing to pay for an allocation. ConstraintAdjustors modify only the demand for, or restriction to, the shows included in the allocation, while AllAdjustors combine the techniques of bid and constraint modification.

Participators that modify their bid amounts, those classified as BidAdjustors and AllAdjustors, do so by one of two increments. They may reenter the auction with an

offer incremented by a percentage of their rejected bid, the rules of the auction establish this minimum increase, see Figure 6.4. Alternatively, our ISCA provides the bidder

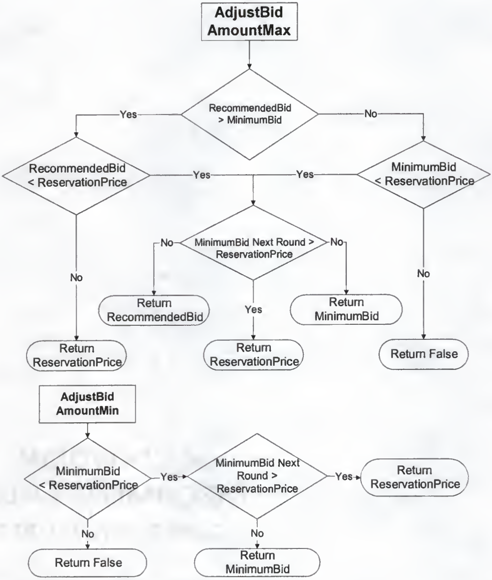


Figure 6.4 Bid Adjustment Flowcharts

with a recommended bid increment to assist in the formation of subsequent bids, agents may increase their prior bid by this recommended amount. The recommended increment is calculated as the difference between the amount the individual bid previously and the seller's discounted list price for what was determined to be the best allocation for that

bidder's demographic requirement in that round. Following the rules imposed by our auction, any recommended bid is at least the minimum percentage increase above the previous bid but may be greater. The option to choose between these two increments dictates the bid adjustment strategies, either Max or Min, depicted in Figure 6.4. In either case, prior to posting, a check is made to see if the interval between the prospective bid and the reservation price is less than the minimum bid amount required in the next round. If the gap is smaller than an allowable bid, the bidder will submit a bid equal to her reservation price to avoid a situation where her current bid is less than her reservation price but she can not reenter the bid because her next bid will not meet the minimum increment requirement.

BidAdjustors and AllAdjustors choosing the maximum of the percentage or recommended increase are acting on the assumption that they will have a greater chance of achieving an allocation if they choose the largest increment. This approach could be viewed as representative of risk averse bidders with the difference between the minimum percentage and the recommended increase being the risk premium (Varian, 1992). If the buyer believes that there is a chance that a lower bid will be successful in the subsequent round the minimum increment strategy would be a better choice. Regardless of the strategy chosen, the buyer cannot bid an amount greater than her reservation price.

Constraint adjustments involve relaxing the restrictions imposed on the shows required or allowed to be allocated to the bidder. These restrictions include the bounds on the number of desired shows, the number of shows included in the desired show list and the number of forbidden shows. Figure 6.5 depicts the various modifications that may be made to provide the seller greater freedom in his allocation. As a minimum the

bidder must loosen a restriction by one unit. Bidders are advised of the constraint(s) that cannot be met. For example the shows available to allocate a bidder may not satisfy the minimum number of shows desired by the bidder, in this case the bidder is notified that her lower bound of desired shows is too high. Agents use this information to direct the choice of constraint modification strategy.

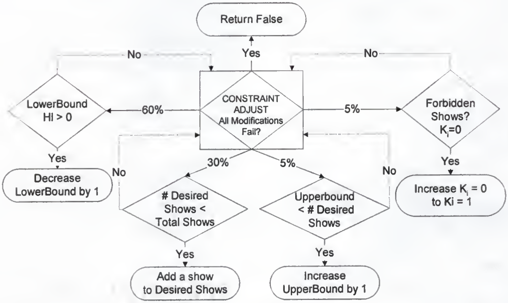


Figure 6.5 Constraint Adjustment Flowchart

One or a combination of the modification strategies is assigned the agent at inception. The percent of agents assigned to each strategy will be varied during experimentation to determine if a particular strategy is dominant. When designated as inactive all bidder types, except the Evaluator, will attempt to reenter the auction following their adjustment strategy until they can no longer meet the minimum increment guidelines.

Figure 6.6 depicts the bidding strategies after the initial bids are placed. Notice that AllAdjustors first attempt to modify the amount they bid and when they have reached a point where they can no longer increase the dollars bid by the minimum increment they resort to modifying their constraints.

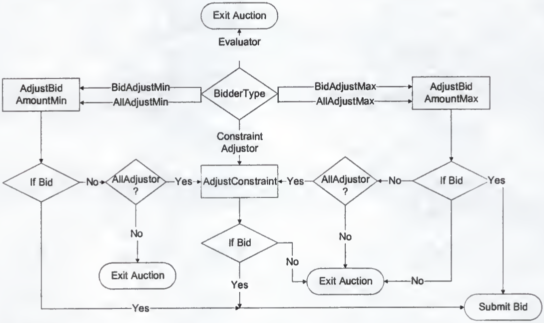


Figure 6.6 Bid Strategy for Rounds > 0 (for inactive bidders only)

6.11 Summary

We created agents to act as bidders in our simulations. The information gathered from the data provided guidelines for establishing the parameters for the total number of bidders, demographic gross rating points, reservation price, number and length of commercials, type product and the number of commercials allowed in each show. The agent's demographic category, number of desired shows, the associated bounds to desired shows, and entry round were randomly assigned from frequencies based on anecdotal evidence about industry practices and justifiable assumptions. Bidding strategies were

founded on the characteristics of bidder types in electronic auctions proposed by Bapna et al. (1998) and the assumption of rational behavior of participants in a competitive environment with information asymmetries. We assume that bidders will accept an allocation as long as it meets the criteria and restrictions specified by the agent in the associated bid. Therefore, we do not include logic permitting bidder rejection of an allocation or early withdrawal from the game.

CHAPTER 7
EXPERIMENTAL DESIGN

7.1 Introduction

The goal of our experiments is to test the auction mechanism's performance under various conditions. The measures used to evaluate the Incompletely Specified Combinatorial Auction's performance are allocative efficiency, the optimality of unit assignment and the length of the auction. The test scenarios include varying the number of each type of bidders, changing the minimum bid increment requirement, applying various stopping rules and modifying the amount of calculation time the heuristic is given in each round. Section 7.2 defines the performance measures used in our analysis, while the subsections of 7.3 set up the parameters for each of the experiments.

7.2 Performance Measures

Historically the performance of auctions has been judged on how well they allocate the product(s) in terms of efficiently and optimality. Additionally, consideration is given to the amount of time it takes to reach a stopping point or equilibrium. We will validate our mechanism using these same measures. We will also study our solution methodology by comparing its results with the solutions obtained by a commercial integer programming package on representative problems that are sufficiently small for it to reach a solution to the combinatorial optimization.

7.2.1 Efficiency

When the seller always sells an object to the bidder with the highest realized valuation for that object, as long as that value is greater than the seller's reservation price, the auction is said to be efficient (Armstrong, 1999). While this measure is fairly straightforward in a single unit environment, complications arise when trying to evaluate efficiency for multi-item auctions, especially those involving package bids. In a combinatorial auction one must consider the impact of overlapping demand for the various items that may limit the feasible allocation of units when determining a mechanism's efficiency. Achieving efficiency has been the goal of many of the recently developed auction mechanisms including the famous Federal Communications Commission (FCC) spectrum license auctions (McMillan, 1994; McAfee & McMillan, 1996; Cramton, 1995). Another reason for the active interest in efficiency is that evidence suggests that efficiency and optimality are complementary in that an inefficient assignment of goods leads to less than optimal seller revenue (Ausubel & Cramtom, 1999).

Establishing an exact measure for efficiency in the multi-unit environment has proven difficult. Krishna and Perry (1998) view efficiency from a social welfare prospective by defining an ex post efficient auction as one were the chosen allocation maximizes social welfare over all types of bidders. Simply stated this means that there is no other allocation that maximizes the sum of the agents' payoffs. Ausubel (1996) provides a procedure for extracting an efficient allocation in a sequential manner. By determining the allocation of goods associated with the highest total bids if Bidder *i* were

absent from the bidding we can determine the marginal surplus that bidder *i* brings to the auction. This process must be repeated for every possible permutation of the number of possible allocations to bidders and thus would become intractable for large problems.

Experimental economics provides an alternative approach that has been effective for research into combinatorial auctions (Plott, 1997). In a laboratory setting information not readily discernable in a true environment can be controlled and manipulated for analysis. For example, in a combinatorial auction in order to determine efficiency the experimenter needs to extract each bidder's value for every possible subset of objects being sold. This information is normally privately held by each bidder. Recent laboratory experiments involving combinatorial auctions used this notion to establish valuations for the possible bundles created by the bidders. Assigning a redemption value $V_b(y)$ to be paid to bidder *b* for holding a combination of objects indicated by *y*, reservation values for the various combinations were established and known to the experimenter. The subjects involved in the study were paid an amount representing the redemption value of the combinations of goods they were ultimately allocated by the auction mechanism. Use of this type of financial incentive is a well documented methodology in experimental economics (Plott, 1997).

With the value information experimenters were able to define efficiency as:

$$E = \frac{\sum_{b=1}^{B} v_b y_b(A)}{V^*},$$

where

$$V^* = \max \sum_{b=1}^{B} v_b y_b$$

subject to the feasibility of the y_b's. The $y_b(A)$'s are the final allocation chosen by the auction (Bykowsky, Cull & Ledyard, 1995; Ledyard, Porter & Rangel, 1997; DeMartini, Kwasnica, Ledyard & Porter, 1999). Notice that efficiency is defined on the set of feasible solutions, rather than over all possible combinations.

In the simulation of our auction we also establish a reservation price for each bidder or agent. The valuations are a step function, where an allocation not meeting the constraints has no value to the bidder and a value equal to a defined reservation price for all allocations satisfying constraints. The items or shows in each bundle are not explicitly valued, but are represented in the valuation by constraints requiring placement in specific shows. To determine the efficiency of our auction we define the final allocation of the auction mechanism as:

$$y_b(A) = \arg\max \sum_{b=1}^{B} a_b y_b .$$

s.t. Constraints (1.1)-(1.11d)

Further we determine the maximum value attainable in the auction as:

$$V^* = \max \sum_{b=1}^{B} v_b y_b$$

s.t. Constraints (1.1)-(1.11d)

From these two equations we can formulate an efficiency measure similar to those used in the prior combinatorial auctions studies as:

$$E = \frac{\sum_{b=1}^{B} v_b y_b(A)}{V^*},$$

Note that this is an estimation of the efficiency as it relies on the outcome of our heuristic that may not achieve an optimal solution to either problem. However, when solving each problem, we end with a range from our final feasible solution to a theoretical upper bound. This gap is often small and is reported in our experiments.

7.2.2 Optimality

A major difficulty with allowing combinatorial bids is the computational difficulty of determining the revenue maximizing set of winning bids. A seller of *n* items could conceivably receive $2^n - 1$ possible bid combinations. The selection of the best combination from among the potentially overlapping bids is a combinatorial optimization problem that is known to be NP-complete (Rothkopf, Pekeč & Harstad, 1998). Thus the investigation of optimality for combinatorial auctions has been relegated to investigating small problems and or problem structures that permit tractability (Armstrong, 1999; Rothkopf, et al., 1998).

Optimal auctions, from the seller's perspective are those that maximize seller revenue. Maximum revenue, in a single item auction, results from allocating efficiently then extracting as much buyer surplus as possible (Myerson, 1981). Ausubel and Cramton (1996) contend that the seller maximizes revenue in a multi-unit auction by allocating objects efficiently subject to exceeding the seller's reservation price. Unfortunately, this notion has yet to be proven for combinatorial auctions. However, the measure of how much buyer surplus the auction is able to extract from the winning bidders is an important consideration in the design of the mechanism. We will report the

percentage of the maximum possible revenue from the final allocation actually captured by the seller as our measure of optimality.

$$\text{Optimality} = 1 - \frac{\sum_{b=1}^{B} (v_b - a_b) y_b(A)}{\sum_{b=1}^{B} v_b y_b(A)},$$

where $y_b(A)$ represents the final allocation of the auction mechanism.

7.2.3 Auction Length

The time it takes for our iterative auction to reach a defined stopping point is an important design consideration. The number of rounds and the time between them will determine the length of the auction. The termination of our auction is based on soft stopping rules, the exact nature of those rules will be varied in the experiments to determine the most effective. There are tradeoffs between auction length and efficiency with an iterative auction format. Increasing the number of rounds provides an opportunity for bid modifications the may enhance the chances of creating packages with better fit and thus boosting efficiency. Lengthy auctions, however, could lead to reduced seller profitability from administrative costs associated with each round or opportunity costs associated with forgone rental revenue of the goods (DeMartini, et al., 1999). In our environment there is a generally agreed upon period over which upfront sales are conducted which suggests the maximum overall length of the auction. There is no required completion time that would dictate that the auction finish by an exact date and time. “Upfront” sales generally last three to four weeks, therefore an auction that doesn’t

reach its goal for months would not be satisfactory. The minimum time between rounds, is influenced by the amount of time required by the heuristic to determine an allocation.

7.2.4 Solution Methodology Performance

Ideally our auction will result in the optimal allocation of goods. Due to the complexity of the problem we have developed a heuristic that achieves a satisficing solution we trust is reflective of the optimal solution. To determine how well our heuristic performs we will compare its results with those obtained by a commercial integer programming package, CPLEX 6.5, on representative problems that are sufficiently small for it to reach a solution to the combinatorial optimization. We will also analyze the amount of time each mechanism takes to compute the solutions as time to completion is important to the performance of our auction.

7.3 The Experiments

Using automated agents to simulate bidders we test the Incompletely Specified Combinatorial Auction (ISCA) on a variety of conditions. To determine the impact of the different bidder types on the efficiency, optimality, and length of the auction the percentages of each type are modified and the results analyzed. We also investigate the influence of altering the minimum bid increment. The effect of the various round lengths is analyzed by adjusting the amount of time given to the heuristic to determine the allocations. Experiments are also conducted to determine the impact of closing rules using four different conventions. Three of the rules are "soft" in that they depend on some activity inherent in the rounds of the auction and can not be determined in advance, while one is determined a priori. Finally, we conduct a comparative analysis of the

heuristic results and the solution to the integer programming problem on a small but representative problem.

7.3.1 Modification of Bidder Types

To analyze the impact of the various bidder types we modify the percentage of bidders in the different types and compare on the performance and results of the Incompletely Specified Combinatorial Auction. Table 7.1 summarizes the experimental parameters for the modification of the participation of the various bidders categories.

Table 7.1 Bidder Strategy Experiment Parameters

	BidAdjustor		**AllAjustor**					
Eval	**Min**	**Max**	**Min**	**Max**	**Constr.**	Bid Increm	Stopping Rule	Round Time
100%	0	0	0	0	0	N/A	Activity	15 min
0	100%	0	0	0	0	5%	Activity	15 min
0	0	100%	0	0	0	5%	Activity	15 min
0	0	0	100%	0	0	5%	Activity	15 min
0	0	0	0	100%	0	5%	Activity	15 min
0	0	0	0	0	100%	5%	Activity	15 min
0	50%	50%	0	0	0	5%	Activity	15 min
0	0	0	50%	50%	0	5%	Activity	15 min
1/6	1/6	1/6	1/6	1/6	1/6	5%	Activity	15 min
5%	20%	20%	20%	30%	5%	5%	Activity	15 min

The first six trials test each bid strategy in isolation while the remainder look at the interaction of the bidding types. In our first experiment we simulate a first price sealed bid auction since an Evaluator bids her reservation price and if rejected does not modify her bid or reenter the auction. The remainder of the experiments involve some form of iteration, similar to a progressive auction. The second and third experiments restrict bid modifications to incrementing the amount bid. The remaining experiments exercise an

option that is fairly unique to the multi-criteria bid, that is the ability to modify the constraints imposed on the allocation by the buyer. Experiment four resembles an iterative first price auction where at each round the requested package is modified but not the amount bid. The fifth and sixth trials incorporate all bid modification strategies, with the sixth experiment representing what we feel is a plausible distribution of bidder types. We use this generic distribution in the remainder of the experiments.

7.3.2 Modification of Bid Increment

Changing the minimum bid increment should impact the amount of time or number of rounds the auction takes to reach its stopping point. The greater the increment the more rapidly bidders will reach their reservation price. The bidder type for all experiments remains constant and reflects the plausible generic distribution across all types. A list of the bid increments tested is provided in Table 7.2. Note that this is a full factorial experiment with all dependent variables modified. Bidder types are consistent with previous experiments. Each minimum increment is tested against each stopping rule. For each activity based stopping rule we also investigate longer calculation times of 30 and 60-minutes to determine if the longer times have any impact. With the other stopping rules will investigate 5 and 15-minute rounds only.

7.3.3 Modification of Stopping Rule

Manipulating the stopping criteria of the auction can have a significant influence on the mechanisms performance. We analyze the effect of four stopping rules. The "activity" stopping rule allows the auction to continue until there are no new bids placed. This is the least restrictive of the rules. The "minimum revenue" rule continues the

Table 7.2 Full Factorial Experimental Design

Eval	**BidAdjustor**		**AllAdjustor**		**Constr.**	**Bid Increm**	**Stopping Rule**	**Round Time**
	Min	**Max**	**Min**	**Max**				
5%	20%	20%	20%	30%	5%	5% 10% 15%	Activity	5 min 15 min 30 min 60 min
5%	20%	20%	20%	30%	5%	5% 10% 15%	Min Revenue	5 min 15 min
5%	20%	20%	20%	30%	5%	5% 10% 15%	Min Revenue & Activity	5 min 15 min 30 min 60 min
5%	20%	20%	20%	30%	5%	5% 10% 15%	# Rounds	5 min 15 min

auction until a pre-specified revenue goal has been achieved. A combination of the two previous rules extends the auction until the allocation attains a revenue goal and there are no new bids. These rules are considered soft stopping rules that promote bidder activity. We also test a hard stopping rule where we designate, in advance, the round in which we will end the auction. We do not reveal the stopping round and the semi-seal format of our auction does not provide signal information from early rounds. Therefore, we assume bidders have no incentive to delay entry into the auction. The full factorial experiment presented in Table 7.2 indicates the parameters upon which the stopping criteria are tested

7.3.4 Modification of Maximum Round Time

The amount of time that the heuristic is allowed to operate in each round will influence the mechanism's performance. The longer the amount of time allotted to the

mechanism to process the bid information the greater the opportunity for the heuristic to find the most efficient fit. Round lengths are specified in Table 7.2, notice that tests look at different combinations of round lengths dependent on the stopping rule. Since the activity based rules continue until no new bids are received their results should provide insight into the results of the other two rules. The decision not to conduct the longer 30 and 60 minute experiments for the Minimum Revenue and Maximum Round stopping rule was made in the interest of conserving experimental time.

7.3.5 Heuristic Methods versus Integer Programming

The solution to the integer program presented in Chapter 3 provides a benchmark against which we compare the heuristic performance. Obviously, since this is a combinatorial optimization the problem must be scaled down to allow the auction to be solved to optimality on a commercial software package. CPLEX 6.5 is the integer program solver used for our study. The base problem solved by both mechanisms consists of 30 bidders competing for placement in 3 shows. The tests are conducted on the same computer to rule out any variability that could result from differing configurations. The test parameters reflect what was determined in the previous tests of the heuristic to achieve the greatest seller revenue. We will analyze the comparative running times of both mechanisms and ultimate solution values. Additionally, we will seed CPLEX with the heuristic's final solution and see if CPLEX can find a better outcome.

7.4 Summary

This chapter defines the performance measures with which we test our Incompletely Specified Combinatorial Auction. The measures are adapted from procedures commonly used in the auction literature. Our experiments have been defined not to test every possible combination of parameters, but rather to provide for a large and fairly representative number of the probable scenarios. We have established comparative tests that will pit our mechanism's performance against that of commercially available integer programming software.

CHAPTER 8
EXPERIMENTAL RESULTS

8.1 Introduction

This chapter presents the results of experiments conducted to determine the behavior and performance of the Incompletely Specified Combinatorial Auction. The experiments consisted of simulating bidding activity under a variety of conditions using artificial agents. Characteristics of the simulated agents are reported in Section 8.2. Section 8.3 analyses the characteristics of the various bidder categories and their impact on auction performance. As discussed in Chapter 7, we define performance using three measures, efficiency, optimality and revenue. The auction's stopping rule, the minimum increment required for subsequent bids, and the time our heuristic is given to determine an allocation in each round are the factors manipulated in our experiments. Sections 8.4, 8.5 and 8.7 provide details of the impact of varying these parameters. We also discuss in Section 8.6 how the seller's choice of discount rate, which establishes his reservation price, affects the auction outcome. Finally, we present a comparative analysis of our heuristic's performance and that of a commercial integer programming package.

8.2 Agent Summary Statistics

By using artificial agents we were able to represent bidder behavior in a hyper rational manner and manipulate various aspect of that behavior to test how the

mechanism reacts to different parameters. Each experiment consisted of 325 randomly generated bidders vying for a limited amount of inventory. As discussed in detail in Chapter 7, the valuations held by the agents were determined from regression equations that were garnered from analyzing real data. We conducted several tests using what we will refer to as the "generic" bidder mix. The generic bidder mix consists of a plausibly representative percentage of bidders in each bidding strategy category. Table 8.1 gives a breakdown of the characteristics of this generic set of bidders. The table presents a summary of two different sets of bidders (i.e., two different sets of agents were generated)

Table 8.1 Generic Bidder Summary Statistics

Generic Bidder Summary Statistics																
	Bidder		Demo 1		Demo 2		Demo 3		Demo 4		Demo 5		Demo 6		Bounded	
Type Bidder	#	%	#	%	#	%	#	%	#	%	#	%	#	%	#	%
Set 1																
Evaluator	15	5%	5	33%	3	20%	1	7%	3	20%	1	7%	2	13%	7	47%
ConstraintAdjustor	16	5%	3	19%	6	38%	1	6%	1	6%	4	25%	1	6%	8	50%
AllAdjustorMax	105	32%	16	15%	18	17%	19	18%	20	19%	21	20%	11	10%	45	43%
AllAdjustorMin	59	18%	14	24%	7	12%	11	19%	7	12%	11	19%	9	15%	24	41%
BidAdjustorMax	59	18%	14	24%	6	10%	8	14%	11	19%	7	12%	13	22%	26	44%
BidAdjustorMin	71	22%	13	18%	10	14%	17	24%	9	13%	9	13%	13	18%	31	44%
Set 2																
Evaluator	16	5%	5	31%	3	19%	1	6%	3	19%	1	6%	3	19%	7	44%
BidAdjustorMin	68	21%	12	18%	11	16%	17	25%	11	16%	6	9%	11	16%	32	47%
BidAdjustorMax	60	18%	15	25%	4	7%	7	12%	12	20%	10	17%	12	20%	30	50%
AllAdjustorMin	64	20%	13	20%	9	14%	13	20%	7	11%	12	19%	10	16%	25	39%
AllAdjustorMax	103	32%	17	17%	14	14%	19	18%	21	20%	20	19%	12	12%	43	42%
ConstraintAdjustor	14	4%	1	7%	7	50%	1	7%	1	7%	3	21%	1	7%	7	50%

with which we tested our auction mechanism. The rows indicate the different bidder types defined in Chapter 6 while the columns provide the distribution over these types. The first two columns are summary statistics that give the total number and percentage of

the indicated bidder type. The following columns indicate the number and percentage of the bidder type evaluating their placement in each demographic category. Finally, the last two columns indicate how many of the bidders in that category have indicated that they must have commercials placed in at least one of their requested shows. We define these type bidders as "bounded" bidders because they have specified a positive lower bound on the number of desired shows.

Evaluators and ConstraintAdjustors represent five percent each of the total number of bidders, while AllAdjustorMin, BidAdjustorMax and BidAdjustorMin hold 18%, 18% and 22% respectively of the total bidder population. The largest category of bidders is the AllAdjustorMax type representing 32% of total bidders. Within each category there are an approximately equivalent number of bidders requiring placement in at lease one of their desired shows. Each bidder type is fairly evenly distributed across the various demographic categories, with any variability attributable to random generation of the agents.

We also let bidders enter the auction in any round with the actual distribution of initial bid rounds for the participating bidders listed in Table 8.2. Each column in the table represents a round of the auction, while the rows give the actual number and percentage of bidders that enter in each round for the 2 sets of agents. Notice that the majority of bidders, 83 and 77%, for each agent set respectively, participate in the auction from the start, while some bidders wait until as late as round 15 to place their initial bid.

Table 8.2 Bidders' Entering Round Distribution

Entry Round Distribution															
Rnd	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
Agent Set 1															
#	255	34	9	6	5	0	5	0	0	3	0	0	0	0	8
%	83%	11%	3%	2%	2%	0%	2%	0%	0%	1%	0%	0%	0%	0%	3%
Agent Set 2															
#	249	36	9	8	4	0	5	0	0	4	0	1	0	0	9
%	77%	11%	3%	2%	1%	0%	2%	0%	0%	1%	0%	0%	0%	0%	3%

8.3 Bidder Type Impact

In Section 6.10 of Chapter 6 we defined six bidder categories based on the bidding strategy employed by the agent to modify a rejected bid. Although we use the generic distribution of bidder types in most of the experiments, it is of interest to analyze the impact that each type and various combinations of types have on revenue, efficiency, optimality, auction length, the amount of unsold inventory and number of winning bidders. A summary of the results is shown in Table 8.3. The top section of the table describes the different distributions tested for the two series of agents. We list several interesting characteristics of the auctions including the maximum revenue and the associated round in which it was achieved, efficiency, optimality and units of unsold inventory at the close of the auction. The table also includes information on the number of winning bidders and the percent of winners that are bounded or have a positive number of required shows.

8.3.1 Summary of Impact of Various Bidder Types

The auction consisting of all ConstraintAdjustors posted the best results. This bidder category was not only able to achieve the highest revenue by also had the highest

Table 8.3 Impact of Differing Bidder Type Distribution

Bidder Type Analysis											
Minutes per Round = 15				**Stopping Rule = Activity**					**Min. Bid Increment = 5%**		
		Bidder Distribution									
Evaluators		100%	0	0	0	0	0	0	0	1/6	5%
BidAdjustors	**Min**	0	100%	0	0	0	0	50%	0	1/6	20%
	Max	0	0	100%	0	0	0	50%	0	1/6	20%
AllAdjustors	**Min**	0	0	0	100%	0	0	0	50%	1/6	20%
	Max	0	0	0	0	100%	0	0	50%	1/6	30%
Constraint		0	0	0	0	0	100%	0	0	1/6	5%
		Results Set 1									
Revenue		25472	23451	20031	26042	24414	27393	22496	25337	24866	24579
Round Achieved		3	22	11	20	16	18	16	22	12	15
Efficiency		100	94.39	86.67	96.04	93.48	100	90.47	95.30	95.62	96.07
Optimality		100	91.83	86.67	98.95	91.83	100	89.84	94.88	94.95	93.62
Unsold Inventory		4	4	49	1	1	1	13	3	0	3
# Winners		119	121	113	134	141	139	120	138	121	128
% Bounded		11%	13%	10%	1%	5%	1%	11%	4%	7%	5%
		Results Set 2									
Revenue		25345	23552	20566	26263	24164	27405	22752	25554	25056	24451
Round Achieved		5	11	12	14	14	14	19	14	20	16
Efficiency		100	97.71	93.07	98.54	92.76	100	93.04	97.84	96.27	95.76
Optimality		100	91.89	88.72	96.95	91.58	100	91.21	94.87	96.43	93.20
Unsold Inventory		3	2	40	2	0	0	9	0	1	6
# Winners		118	110	116	133	139	141	112	132	127	126
% Bounded		9%	15%	18%	2%	4%	1%	13%	3%	9%	5%

number of winning bidders, had minimal or no unsold inventory and 100% optimality and efficiency. These results were achieved in as little as 14 rounds, a little later than some of the other experiments. The data show that auctions consisting of only AllAdjustors also performed well with regard to revenue, allocations to a consistently high number of bidders while leaving very little inventory unsold. Evaluators also faired well posting the second highest achieved revenue, 100% efficiency and optimality, with only one or two units of unsold inventory. BidAdjustors under-perform in as much as

they leave significant amounts of inventory unsold, achieve the least amount of revenue, select the fewest winners and function worst in terms of efficiency and optimality.

Combining the various bidder types produces results that reflect both extremes. We analyze a uniform distribution across the different types as well as the "generic" distribution that is used for the majority of the experimentation in this study. The outcome for the combination of bidder types is weaker in all aspects than of the set of bidders willing to modify their constraints such as when the auction consists of only ConstraintAdjustors or AllAdjustors. However, in comparison to the results obtained from the bidder sets consisting on only BidAdjustors, or the inflexible types, the combination of bidder categories obtained better performance measures.

8.3.2 Analysis of Bidder Type Results

The characteristics of ConstraintAdjustors, namely that their initial bid represents their reservation price and their willingness to modify their constraints to achieve an allocation, highlight the fact that flexible bids are more likely to be successful. This notion is further supported by the data from the experiments consisting of AllAdjustors (Min and Max) either individually or in combination. In contrast, as might be expected, the more inflexible bidders or those bidders who will only modify their bid amount and refuse to compromise their show selections, such as BidAdjustors, are not as effective.

Based on the above discussion it would seem that an auction consisting of only Evaluators should have poor performance characteristics because this bidder type is completely inflexible, allowing for no modifications. However, when all 325 bidders are of this type there are enough unbounded Evaluators, those not requiring specific show

placements, to consume the available inventory. Since, Evaluators bid their true valuations, unconstrained Evaluators mimic the characteristics of ConstraintAdjustors that have eliminated all their demands and therefore display the favorable characteristics indicated in Table 8.3.

The true mix of bidder types in the actual environment in which the auction is deployed will influence the performance of the auction. The above results suggest that a higher the percentage of flexible auction participants will promote greater efficiency and optimality as well as increased revenue.

8.4 Bid Increment Impact

To determine the impact of various minimum bid increases on the Incompletely Specified Combinatorial Auction's performance and establish a rule to apply to our mechanism we conducted experiments using three distinct increments, 5%, 10% and 15%. Appendix C Tables C1-C18 give detailed round by round descriptions of the performance measures for each combination of minimum bid increment, stopping rule and computing time. A summary of those results is included in Tables 8.4-8.6 presented below. The individual cells in each of the three tables, one table for each performance measure, represent the outcome for the combination of dependent variables. For example, the first cell of Table 8.4 indicates the revenue for an auction using the activity stopping rule with a 5% minimum bid increment that is allowed to calculate for 5 minutes is 24391.70.

8.4.1 Analysis of Bid Increment Results

Our analysis shows that varying the minimum required percentage increase for subsequent bids does have an impact on the performance of the auction. Because a larger increment requires the bids to escalate more quickly, the minimum revenue goal is achieved faster. In fact in our experiments the minimum revenue was achieved in round 5 or 6 with a 5% minimum, round 4 when a minimum 10% increase was required and as early as round 3 with a 15% increment. However, the data in Table 8.4 also shows that when the increment was highest, for example when the minimum increase required was 15%, the maximum overall revenue, in most cases, actually decreased from that achieved with a 10% increment when the auction was allowed to achieve equilibrium

Table 8.4 Maximum Revenue Summary

Revenue (000)						
Set 1						
Stopping Rule	Percent = 5%		Percent = 10%		Percent = 15%	
	5 Min	15 Min	5 Min	15 Min	5 Min	15 Min
Activity	24391.70	24578.50	25264.10	25052.10	24979.40	24818.50
MinRev	23146.70	23487.20	23538.10	23617.70	23377.90	23284.90
MaxRound	24387.60	24522.30	24919.10	25143.20	24869.20	25069.40
Set 2						
Stopping Rule	Percent = 5%		Percent = 10%		Percent = 15%	
	5 Min	15 Min	5 Min	15 Min	5 Min	15 Min
Activity	24409.40	24334.10	24641.60	24588.60	23593.50	24849.00
MinRev	23498.80	23203.60	23586.20	23498.30	23298.00	23395.00
MaxRound	24287.20	24138.00	24467.90	24370.70	24923.30	24646.20

using the activity stopping rule. We attribute this to the fact that the agents in our study are budget constrained and a high minimum increment forces more bidders out of the auction early because they have reached they reservation price, thus decreasing

competition by reducing the bidder pool. The highest overall revenue was obtained for an auction that required a 10% minimum increase and used the activity stopping rule.

The greatest optimality was achieved using the highest bid increment. Generally, the auction is able to extract the most surplus from the winning bidders where those that required a 15% bid increment, see Table 8.5 for the details. The conditions generating the highest optimality were the Activity stopping rule with a 15% minimum bid increment that is allowed to calculate for 5 minutes.

Table 8.5 Summary of Optimality Results

Optimality						
Set 1						
Stopping Rule	Percent = 5%		Percent = 10%		Percent = 15%	
	5 Min	15 Min	5 Min	15 Min	5 Min	15 Min
Activity	93.4445	93.6227	95.5487	95.2880	96.2838	95.4169
MinRev	91.0156	92.4770	93.9426	94.0618	93.3870	93.7843
MaxRound	93.4635	93.8447	95.3553	95.4414	95.7468	95.7616
Set 2						
Stopping Rule	Percent = 5%		Percent = 10%		Percent = 15%	
	5 Min	15 Min	5 Min	15 Min	5 Min	15 Min
Activity	93.2974	93.3839	94.0773	94.3596	94.7172	94.6865
MinRev	92.3043	91.9155	93.4855	93.3097	92.9948	93.2119
MaxRound	93.9622	92.6800	94.3917	93.9918	94.9689	94.3543

The most efficient auction, or the auction that most effectively assigned units to the bidders that value them the most, was not as consistent in terms of the impact of the minimum bid increment. Table 8.6 indicates that when the auction was allowed to continue for several rounds, as in the case of those using the activity and maximum round (10) stopping rules, the higher percentage increases forced higher efficiency. This could be because by requiring a higher subsequent bid the bidders' true valuations surfaced earlier, thus giving the mechanism the opportunity to assign units to those with the

Table 8.6 Summary of Efficiency Results

Efficiency						
Set 1						
Stopping Rule	**Percent = 5%**		**Percent = 10%**		**Percent = 15%**	
	5 Min	**15 Min**	**5 Min**	**15 Min**	**5 Min**	**15 Min**
Activity	94.7058	96.0650	97.4008	96.5083	96.9639	96.7413
MinRev	89.4526	90.8648	91.5521	91.8615	98.0020	91.2745
MaxRound	93.9347	94.8273	95.0520	95.7361	95.7839	97.3613
Set 2						
Stopping Rule	**Percent = 5%**		**Percent = 10%**		**Percent = 15%**	
	5 Min	**15 Min**	**5 Min**	**15 Min**	**5 Min**	**15 Min**
Activity	96.4054	95.5788	95.2012	95.9128	97.3315	94.9535
MinRev	91.6006	90.6344	91.5030	91.3863	97.4600	91.8082
MaxRound	93.1122	92.5738	94.0633	94.1339	95.6039	94.3855

highest revealed valuations. However, in the case of the minimum revenue stopping rule the 15% the results were inconsistent between the 10% and 15% increments. Minimum revenue was achieved in round 3 under these parameters and one could speculate that the small number of rounds did not give an opportunity for those bidders with the highest true valuations to become the dominant bidders. Due to the shortness of the auction equilibrium was not achieved.

Still another aspect of the auction that is influenced by the minimum increment percentage is the number of rounds needed to achieve equilibrium. Tests showed that on average when the activity stopping rule was employed auctions requiring only 5% increments needed an average of approximately 22 rounds to achieve equilibrium, 10% minimum increase auctions also finished in around 20 rounds while those requiring at least 15% added to previous unsuccessful bids concluded in 19 rounds. Figure 8.1 shows the revenue progression for 15-minute calculation times. Also notice that the higher the

required increment the faster the initial revenue gain, however the 10% increment eventually achieves the highest revenue.

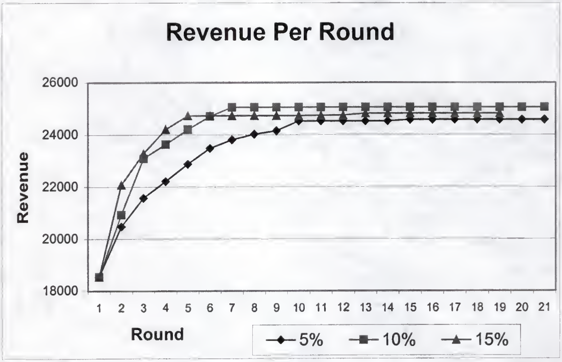


Figure 8.1 Revenue Attained and Stopping Round For Each Percentage Increment

We tested the significance of the impact of the percentage increase on the independent variables, efficiency, optimality and revenue to confirm the conjecture presented above. The results support our claims that the minimum increment required does effect the performance of the auction. The results of the Multivariate Analysis of Variance are presented in Figure 8.2 below. The P-value of .0001 for all tests indicates that the test is significant and we can therefore reject the hypothesis that there is no overall percentage increase effect and conclude that the various levels of minimum bid

increments do impact the performance measures of the Incompletely Specified Combinatorial Auction.

Manova Test Criteria and F Approximations for the Hypothesis of no Overall PCT Effect
H = Type I SS&CP Matrix for PCT E = Error SS&CP Matrix

S=2 M=0 N=7

Statistic	Value	F	Num DF	Den DF	Pr > F
Wilks' Lambda	0.08968056	12.4761	6	32	0.0001
Pillai's Trace	1.34202819	11.5580	6	34	0.0001
Hotelling-Lawley Trace	5.33683854	13.3421	6	30	0.0001
Roy's Greatest Root	4.18717260	23.7273	3	17	0.0001

Note: F Statistic for Roy's Greatest Root is an upper bound.
Note: F Statistic for Wilk's Lambda is exact.

Fig 8.2 Manova: Percentage Effect on Auction Performance

Although, evidence suggests that increments of 10% or 15% achieve the best results for our mechanism it is important to acknowledge the psychological impact of the larger bid increases. Within the environment represented in this study the bid amounts correspond to hundreds of thousands, even millions of dollars. The difference between a 5% and 10% minimum increment could be significant. The decision on minimum bid increment should be tempered by a consideration of the scale of the amounts to be tendered.

8.4.2 Summary of Bid Increment Findings

The results of our experiments show that modifying the minimum bid increment does impact the efficiency, optimality, realized revenue and length of the auction. A 10% minimum increment coupled with either the activity or maximum round stopping rule

yields the best results in terms of revenue and efficiency, while optimality is maximized when using the largest increment. If brevity of the auction is important a larger required increase facilitates early attainment of a predefined revenue goal.

8.5 Analysis of Stopping Rule Effect

Four stopping rules were employed in the experiments to determine the performance characteristics of the Incompletely Specified Combinatorial Auction. They are the Activity Rule that resembles the progress of a classic English auction allowing the auction to continue until no further bids are submitted. To imitate an English auction where the seller has a reservation price we designed a stopping rule that combined the Activity rule with a requirement to acquire a predetermined minimum amount of revenue. As it turns out in our setting the minimum revenue was obtained in each of the auctions using the activity rule. Thus the outcome is the same for both categories and we do not duplicate the results. The two other stopping rules terminate the auction when a predetermined revenue goal has been reached or a designated round completed.

Evidence from the experiments conducted in this study indicates that the choice of stopping rule does indeed influence the mechanism's performance characteristics. Tables 8.4-8.6, presented in the previous section provide details on the variability of the performance measures between rules. As one might expect, the activity rule realizes the best over all performance measures as it allows the auction to achieve a natural equilibrium. While the minimum revenue rule stops accepting bids even though there may be bidders willing to participate, thus extracting the least surplus and possibly inefficiently allocating units. The Manova results confirm the significance of the effect

of the stopping rule. P-values in Figure 8.3 equaling 0.0001 for all tests indicate that we can reject the hypothesis of no overall stopping rule effect and conclude that differing the stopping rule for the auction impacts its performance characteristics.

Manova Test Criteria and F Approximations for the Hypothesis of no Overall RULE Effect
H = Type I SS&CP Matrix for RULE E = Error SS&CP Matrix

S=2 M=0 N=7

Statistic	Value	F	Num DF	Den DF	Pr > F
Wilks' Lambda	0.040998950	21.0065	6	32	0.0001
Pillai's Trace	1.40614602	13.4177	6	34	0.0001
Hotelling-Lawley Trace	12.4846154	31.2115	6	30	0.0001
Roy's Greatest Root	11.53949096	65.3904	3	17	0.0001

Note: F Statistic for Roy's Greatest Root is an upper bound
Note: F Statistic for Wilk's Lambda is exact

Fig 8.3 Manova: Rule Effect on Auction Performance

8.5.1 Trickle Effect

While analyzing the data on auctions using the activity rule we noted a phenomenon that we refer to as the Trickle Effect. In a number of the experiments the bidding continued for several rounds yet there was no improvement in revenue (specifics are provided in Appendix C, Tables C1-C12). The auction did not terminate since, during each round, there was at least one revised bid submitted. A more detailed look at the data indicated that the majority of revisions were being made to constraints, for example decreasing the number of shows required. Due to the nature of the multi-criteria bid, a bidder could conceivably enter a highly restrictive low bid in the initial rounds and continue making modifications to constraints and or her bid amount for an extended

number of rounds. There may be some merit to this approach; the bidder could be attempting to obtain signal information from the recommended bids provided by the mechanism. Further study is warranted into this particular bidding strategy.

This type of bidding could prolong the auction beyond a natural stopping point. We did not limit how the constraints were modified by our agents, allowing any agent to modify their constraints to completely eliminate all restrictions given enough rounds. The percentage of winning bidders that ended the auction with a positive lower bound on the number of required shows decreased with the length of the auction. The maximum percentage of constrained bidders in any auction was 24%, with most auctions ending with only 2% to 7% of bidders constrained. This is much less than the 44% of original bidders requiring placement in at least one show.

8.5.2 Summary of Stopping Rule Influence

The choice of stopping rule has been shown to impact the performance of the auction mechanism. The activity stopping rule was the overall best choice to achieve the highest revenue, optimality and efficiency. However, this rule may potentially prolong the auction beyond a reasonable number of rounds due to the Trickle Effect, the ability of bidders to make minor modifications to their bid by modifying the different restrictions conveyed by the multi-criteria combinatorial bid.

8.6 Influence of Seller Reservation Price

The minimum revenue goal established for our experiments was calculated by summing the discounted list prices for the units available for sale. These discounted list prices are used throughout our solution method in various functions necessary to

determine allocations to bidders. Television Network management establish this discount based on anticipated market demand thus it may vary from the 55% employed in our experiments. To determine if varying the rate had a significant impact on our auction performance we tested discount rates of 45, 50, 55, 60 and 65 percent. The experimental parameters consisted of the activity stopping rule, a 10% minimum bid increment and 15 minute calculation time. Table 8.7 presents the results of the experiments. Each column

Table 8.7 Analysis of Discount Rate Impact

Discount Rate Analysis					
Stopping Rule = Activity Minutes per Round = 15 Min. Bid Increment = 10%					
Discount Rate	**45%**	**50%**	**55%**	**60%**	**65%**
Revenue Goal	28260	25691	23122	20552	17983
Revenue	21308	24848	25052	24933	24890
Round Achieved	16	12	7	7	16
Efficiency	93.51	99.23	96.51	100.00	96.95
Optimality	95.85	96.93	95.29	95.13	95.01
Unsold Inventory	100	13	5	2	2
# Winners	110	121	124	131	121
% Bounded	4%	9%	7%	9%	4%
Set 2					
Revenue Goal	28260	25691	23122	20552	17983
Revenue	20998	24662	24589	24344	24502
Round Achieved	13	16	12	11	11
Efficiency	96.05	94.95	95.91	95.94	96.13
Optimality	96.45	95.78	94.36	94.15	93.37
Unsold Inventory	107	16	3	2	0
# Winners	111	124	124	121	127
% Bounded	3%	2%	3%	6%	7%

presents data on each of the different discount rates. We have included statistics on the performance measures, revenue, efficiency and optimality as well as other parameters of interest. The data reveals that at the extremes, such as with a discount rate of 45 or 65 percent the realized revenue is less than for the more moderate discount rates. The most

efficient allocations are achieved when the discount rate is either 50 or 60%, while optimality steadily decreases as the discount rate increases.

Using an Anova we tested the significance of the impact of varying the discount rate on the performance measures. The results presented in Table 8.8 indicate that the

Table 8.8 Analysis of Variance for Various Discount Rates

Analysis of Variance Procedure					
Dependent Variable:		**EFF**			
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	4	11.10296	2.77574	0.66	0.6478
Error	5	21.143	4.2286		
Corrected Total	9	32.24596			
	R-Square	C.V.	Root MSE	EFF Mean	
	0.344321	2.130541	2.056356	96.518	
Source	DF	Anova SS	Mean Square	F Value	Pr > F
DISCOUNT	4	11.10296	2.77574	0.66	0.6478
Dependent Variable:		**OPT**			
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	4	7.41146	1.852865	2.99	0.1304
Error	5	3.0987	0.61974		
Corrected Total	9	10.51016			
	R-Square	C.V.	Root MSE	OPT Mean	
	0.70571	0.82665	0.78723567	95.232	
Source	DF	Anova SS	Mean Square	F Value	Pr > F
DISCOUNT	4	7.41146	1.852865	2.99	0.1304
Dependent Variable:		**REV**			
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	4	20479917.4	5119979.35	60.77	.0002
Error	5	421265.00	84253.00		
Corrected Total	9	20901182.40			
	R-Square	C.V.	Root MSE	REV Mean	
	.979845	1.208797	290.2636732	24012.60	
Source	DF	Anova SS	Mean Square	F Value	Pr > F
DISCOUNT	4	20479917.4	5119979.35	60.77	.0002

discount rate does make a significant impact on revenue and to a lesser degree optimality, but we could not conclude that there is a difference in efficiency. The tests indicate that the choice of discount rate is important and should be based on the ultimate goal of the

seller. If the seller's goal is to have the most optimal allocation the evidence suggests that lower discount rate should be applied. However, to obtain the greatest revenue a moderate discount is the obvious choice.

Special consideration should be given if the closing rule of choice is the attainment of a minimum revenue goal. In our experiments, the minimum revenue goal was determined as a sum of the discounted list for the units available, a larger discount rate will promote the auction reaching the stated goal. If the discount rate is too low an auction using the Minimum Revenue stopping rule may never terminate. Notice in Table 8.7 only those auctions with at discount rate of 55% or greater were able to achieve their revenue goal.

8.7 Heuristic Performance: Changing Computing Time

8.7.1 Standard Heuristic Performance

The final controlled variable in our experiments is the calculation time given our heuristic to determine an allocation. We experimented with four periods, 5, 15, 30 and 60 minutes. The preliminary results from tests using the activity stopping rule indicated that there was no significant improvement from the longer, 30 and 60 minute, runs. Therefore, in the interest of time we tested the remaining closing rules using only 5 and 15 minute calculation times. Refer back to Tables 8.4-8.6 for a summary of the impact on the performance measures from altering the calculation time. We report the results of the 30 and 60 minute calculation times in Appendix C, Tables C3, C4, C7, C8, C11 and C12.

It was anticipated that the longer the mechanism was given to determine an allocation the better that allocation. However, at first glance it would appear that on occasion the additional calculation time had a negative effect on the performance parameters. This result seemed counterintuitive and after further investigation we were able to attribute the occasional reduced performance to a phenomenon commonly found in nature and used in simulated annealing search methods. Annealing is a process in which physical substances are melted and then gradually cooled until some solid state is reached. The process is very sensitive to the rate at which it is cooled, a rate that is too rapid or slow will negatively affect the quality of the final product. Giving our mechanism a longer calculation time allows it to possibly find allocations for more bidders thus they become winners in that round and are not required to modify their bids to be considered in the next round. Although, the auction performs better in initial rounds, improvement in later rounds is reduced because the broader early allocations reduce the ability of the mechanism to extract buyer surplus.

The Manova results presented in Figure 8.4 indicate that the time given to the mechanism to determine an allocation is not a significant factor influencing performance characteristics. All four significance tests give an F value of .2503 and a P-value of 0.0963 which are not significant therefore we can not reject the null hypothesis that calculation time has no overall effect on the independent variables efficiency, optimality and revenue.

When time is coupled with percentage we see that there is a significant impact on the auction's performance. Table 8.5 shows that all tests are significant at the 95% level

Manova Test Criteria and F Approximations for the Hypothesis of no Overall TIME Effect
H = Type I SS&CP Matrix for TIME E = Error SS&CP Matrix

S=1 M=0.5 N=7

Statistic	Value	F	Num DF	Den DF	Pr > F
Wilks' Lambda	0.68065413	2.5023	3	16	0.0963
Pillai's Trace	0.31934587	2.5023	3	16	0.0963
Hotelling-Lawley Trace	0.46917495	2.5023	3	16	0.0963
Roy's Greatest Root	0.46917495	2.5023	3	16	0.0963

Figure 8.4 Manova Time Effect on Auction Performance

and we can reject the hypotheses that there is no overall effect of the combination of percentage and time on the performance of our auction. Similar results are obtainedfrom the interaction of time with the other independent variables indicating a need to consider the amount of time given the mechanism to determine an allocation.

Manova Test Criteria and F Approximations for the Hypothesis of no Overall PCT*TIME Effect
H = Type I SS&CP Matrix for PCT*TIME E = Error SS&CP Matrix
S=2 M=0 N=7

Statistic	Value	F	Num DF	Den DF	Pr > F
Wilks' Lambda	0.40757782	3.0206	6	32	0.0187
Pillai's Trace	0.59442881	2.3965	6	34	0.0487
Hotelling-Lawley Trace	1.44859584	3.6215	6	30	0.0080
Roy's Greatest Root	1.44518915	8.1894	3	17	0.0014

Note: F Statistic for Roy's Greatest Root is an upper bound
Note: F Statistic for Wilk's Lambda is exact

Figure 8.5 Manova Percent*Time Effect on Auction Performance

8.7.2 FastMode Heuristic Performance

The solution heuristic engaged by the auction to determine allocations to bidders incorporates a variety of techniques that are described in detail in Chapter 5. Two of the approaches taken are to solve an aggregate subproblem exactly using dynamic programming or employ a heuristic based on linear programming relaxations referred to as FastAP. FastAP is used during the breadth first search phase of our branch and bound procedure and for 40% of the time spent in the depth first search, the remaining solutions at each branch are determined using dynamic programming. To gain a better understanding of the contribution of FastAP on the heuristic's performance we modified the solution methodology to utilize only FastAP. That is, no exact dynamic programming methods were employed. An Anova analysis comparing the results of the modified heuristic with those obtained from the original formulation indicates that there is no significant difference between the two methodologies. The low F-Values and high probabilities shown in Table 8.9 for each of the independent variables support the conclusion that there are no differences between the performance measures for the two heuristics.

8.8 Heuristic Performance Versus Integer Program

The solution to the integer problem presented in Chapter 3 acts as a benchmark against which we compare our heuristic's performance. To facilitate achieving a solution using a commercial integer programming solver, CPLEX 6.5, the problem had to be

Table 8.9 ANOVA Comparing FastAP to Original Heuristic

Analysis of Variance Procedure					
Dependent	Variable:	**EFF**			
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	2	4.70523180	2.35261590	0.40	0.6706
Error	51	297.87654976	5.84071666		
Corrected Total	53	302.58178156			
	R-Square	C.V.	Root MSE	EFF Mean	
	.015550	2.564011	241675747	94.25691852	
Source	DF	Anova SS	Mean Square	F Value	Pr > F
GROUP	2	4.70523180	2.35261590	0.40	0.6706
Dependent	Variable:	**OPT**			
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	2	6.35720916	3.17860458	2.03	0.1416
Error	51	79.80139247	1.56473319		
Corrected Total	53	86.15860163			
	R-Square	C.V.	Root MSE	OPT Mean	
	0.073785	1.328598	1.25089296	94.15135741	
Source	DF	Anova SS	Mean Square	F Value	Pr > F
GROUP	2	6.35720916	3.17860458	2.03	0.1416
Dependent	Variable:	**REV**			
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	2	1366473.73778	683236.86889	1.46	0.2420
Error	51	23879297.42221	468221.51808		
Corrected Total	53	25245771.15999			
	R-Square	C.V.	Root MSE	REV Mean	
	0.054127	2.814713	684.26713941	24310.36667	
Source	DF	Anova SS	Mean Square	F Value	Pr > F
GROUP	2	1366473.73778	683236.86889	1.46	0.2420

scaled down considerably from the size represented in the previous experiments. The reduced problem consists of 30 bidders vying for 104 units distributed across 3 shows. We chose to compare the results from one round of activity. The integer problem was generated using the parameters of the third round of bidding from an auction using 5 minutes of calculation time, 10% minimum bid increments and the activity stopping rule. The number of active bidders in round 3 had been reduced to 12 from the original 30. With the above parameters the Incompletely Specified Combinatorial Auction was able to

complete the entire auction consisting of 5 rounds in a total of 15 minutes. The duration of the auction includes time used to generate agents, calculate upper bounds, output MPS formatted version of the problem for CPLEX at each auction round, output all solution files for possible CPLEX usage as well as determine the appropriate allocation of units in each of the 5 rounds. The ISCA found a solution that produced revenue of $1728.31 for the seller with a gap of 0.038198%.

An integer problem representing a single round, round 3, of the auction was entered into CPLEX. After over 43 hours the program was unable obtain a single feasible integer solution. We then seeded CPLEX with the solution obtained by our mechanism for round three. CPLEX confirmed that the solution was indeed feasible and after 24 hours was unable to improve on the solution. The initial gap reported by CPLEX for this solution was 43.74%.

8.9 Summary

This chapter presented the results of experiments testing the performance of the Incompletely Specified Combinatorial Auction. We have shown that the mechanism is efficient, the efficiency range for all experiments was 86.67 to 100%. The mechanism also scored well with respect to optimality with a minimum optimality of 86.67%, recorded for an auction consisting entirely of inflexible bidders or those not willing to adjust their constraints, and a maximum of 100% optimality. The experiments proved that utilizing the Incompletely Specified Combinatorial Auction a seller can not only achieve a desired revenue but could conceivably exceed the desired goal when the activity or maximum round stopping rules are employed. Our analysis also provided

evidence that the choice of minimum bid increment and stopping rule affect the performance of the mechanism measured in terms of efficiency, optimality, realized revenue and auction length in terms of the number of rounds required to achieve an equilibrium stopping point. A combination of the activity stopping rule and a 10% minimum bid increment posted the best results. Varying the amount of time, between 5 and 60 minutes, that the heuristic is given to determine an allocation impacted the performance of the mechanism when couples with the other independent variables. Use of the heuristic solution methodology allows us to find a solution to a complex combinatorial optimization problem that is too large to be accommodated by the current state of the art integer program solver, CPLEX 6.5.

CHAPTER 9
CONCLUSION

9.1 Introduction

This chapter contains a discussion of the conclusions garnered from this study as well as proposed directions for future research. We originally posed the question of whether or not an auction mechanism could be designed to accommodate the complexities of a negotiated environment, namely the need for non-specific multi-dimensional bids to guide rather than dictate allocations. In previous chapters we have explored the design issues and reported the results of simulated testing of the prototype mechanism in Chapter 8. In Section 9.2 we discuss these results and draw conclusions about the efficacy of the mechanism. Additionally, limitations of the study are presented to qualify the outcomes in Section 9.3. We conclude the chapter by offering directions for future research into this type of mechanism, including model revisions and experimentation using human subjects.

9.2 Project Overview

This research was motivated by the need for a new auction mechanism that can accommodate buying needs not being fulfilled by current designs. We identified the characteristics missing from current auctions mechanisms that were necessary for an auction to replace or enhance a negotiated environment and made provisions to

incorporate them into our mechanism. A representative negotiated environment was identified, that of television network sales, and described. We described a simple bid structure and modeled an allocation procedure by an integer program. Our auction mechanism, dubbed the Incompletely Specified Combinatorial Auction, consists of multiple rounds of bidding and allocation. Bid modification rules, stopping rules and other features of our auction were also developed. A heuristic was designed to facilitate attaining a satisficing solution to the complex combinatorial optimization problem that is needed to determine product allocations and winner selection.

A simulated auction environment was chosen to test the performance characteristics of our auction mechanism. From data supplied by a representative television network we were able to extract patterns that were used to determine product characteristics such as inventory levels and list prices. We were also able to model buying patterns, including the number of units desired, demographic requirements and associated reservation prices. Using this information we generated artificial agents to act as participants in our simulated auction. Experiments were conducted to discover how well the mechanism performed in terms of efficiency, optimality and realized revenue.

9.3 Conclusions

This research has presented evidence that an auction mechanism can indeed be developed to accommodate the special needs of a negotiated environment. The mechanism modifies a combinatorial auction to accept bids that are incompletely specified in that they provide guidelines to direct rather than dictate the allocation of goods. The mechanism also gives the bidders the ability to impose restrictions on the

allocation through specifications of the multi-criteria package bid. Experiments establish the mechanism as efficient, optimal and revenue maximizing. Analysis of the heuristic developed to accommodate the large problem dimensions indicates that the solutions are optimal or near optimal. As important, the time it takes the heuristic to reach a satisficing solution is minimal and well within the limits imposed by the real-world environment.

9.4 Limitations

This research presents a preliminary investigation into a new auction mechanism. The nature of the investigation has important limitations. First, in order to gain insight into the efficacy of the mechanism we have limited our experiments to simulation with artificial agents. We acknowledge the fact that all nuances inherent to human behavior cannot be adequately programmed into agents. However, it should be noted that artificial agents are being used extensively to assist bidders with various online auctions and their hyper-rational behavior has been favorably compared to Economist's game theoretic model of participant interaction (Varian, 1995). In this study we do not address various aspects of auction theory that should be reviewed before this mechanism is deployed. These issues include buyer trust of the mechanism, potential for bidder collaboration, and other game theoretic characteristics such as more sophisticated bidding strategies.

Our model accommodates the sale of a week of airtime and we make the simplifying assumption that buyers' campaigns are continuous and thus the purchase pattern will repeat from week to week throughout the year. In reality, buyers will want to "flight their campaigns," or place advertising only during specific weeks or days of the year. In order to accommodate flighting our mechanism will need to be expanded to 52

weeks and accept buyer specified airdates. Extending our simple bid structure to 52 weeks will be challenging.

Finally, our mechanism is designed to assist or replace a negotiated environment, yet we have not attempted in this study to analyze the business process changes that would be necessary to facilitate the transition. We anticipate, especially in the television industry, that conversion from negotiation to a business-to-business electronic auction may be met with a great deal of resistance.

9.5 Direction for Future Research

As indicated earlier, this is a preliminary study of our mechanism and as such there is a great deal yet to discover about the mechanism's use as well as new design issues to explore. This section will propose a variety of related subjects that have yet to be considered regarding the mechanism presented in this study.

9.5.1 Expansion of the Mechanism

The mechanism was designed to allocate a week's worth of airtime with the assumption the buyers campaigns are continuous throughout the year and the allocation would be consistent across all 52 weeks. This simplifying assumption fails on two accounts. First, continuous campaigns that last an entire year are rare. Buyers are more likely to want their advertising placed in specific weeks that correspond to dates of other media purchases such as radio and magazine. These active weeks are routinely followed by periods of inactivity or a reduction in their exposure requirements. Additionally, by assuming that each week is a duplicate of the generic week that is auctioned we do not allow for variations in shows throughout the year. Networks routinely schedule special

programming such as sporting events or mini-series whose demographic exposures and costs are not necessarily equivalent to the shows they replace. A classic example would be the Superbowl which commands premium prices. Accommodating these nuances further supports the need to investigate how to expand the auction to encompass 52 weeks of inventory.

The current design does not allow the bidder to reject an allocation. It can be argued that since the units assigned are not necessarily completely specified by the buyer, due to the inexact nature of the bid, that the buyer should have the option to reject an individual allocation. Currently, the only recourse the bidder has is to submit a bid with a higher bid amount or reduced restrictions and hope for a better allocation in subsequent rounds. Expanding the design to a double-sided auction would provide the necessary bid-offer format to facilitate the bidder's rejection of an unacceptable allocation.

Although the mechanism was designed to meet the needs of a particular industry, namely the television industry, it can be generalized to other areas. Not all industries are as protective of their pricing and inventory information, therefore an obvious model modification would be to design an open format that provides signal information to buyers.

Generalizing the ISCA concepts to other environments would require identifying the decision variables associated with the purchase of products. An example could be the purchase of component parts for a manufacturing process. These type purchase decisions are often dependent on delivery options, financial terms and conditions as well as product characteristics and mixes. The bidder would need to specify the various parts, the number desired in total, a specific delivery date and financial considerations such as

interest rate. Flexibility could be incorporated into the bid by establishing bounds on delivery date and or product requirements. These constraints would then be modeled in the allocation engine of the auction mechanism. Identifying problem domains where ISCA might be applicable is an area for future research.

9.5.2 Empirical Investigations

One of the limitations of this research is the use of simulation as the sole means of testing the mechanism's performance. An empirical study using human subjects would supplement our understanding of the effectiveness of the auction mechanism. Several issues, such as human computer interaction, trust, bidding strategies and collusion, can be explored empirically. Establishing bidder trust in a semi-sealed auction has, to our knowledge, not been investigated yet could be a defining issue on the mechanism's acceptance. The fact that the mechanism suggests bids to inactive buyers implies that the mechanism must be considered trustworthy if bidders are to act on the information provided. Another question about the semi-sealed format is its impact on collusion. Sealed-bid auctions are less susceptible to collusion than are open auctions because actual bids are not displayed allowing bidders to deviate from collusive pricing agreements without fear of detection (Mead, 1967; Milgrom, 1987b). It would appear that the same results would hold for the semi-sealed format, human studies could be designed to investigate this phenomenon.

Expanding the bid criteria exacerbates the complexity associated with combinatorial auctions. Investigating the interface design and display information is

another important aspect of the mechanism design that would also benefit from human subject studies.

9.5.3 Game Theory Studies

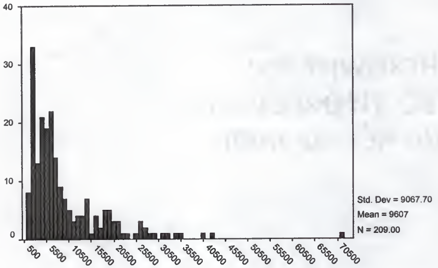
Another extension of this research would be to explore the impact of various bidding strategies. Questions such as the most advantageous time for a bidder to enter the auction and whether or not to bid one's true valuation upon entry have yet to be answered. This study briefly looked at various bidder types and attempted to model their expected behavior, however further analysis into the distribution of these types is warranted.

9.6 Summary

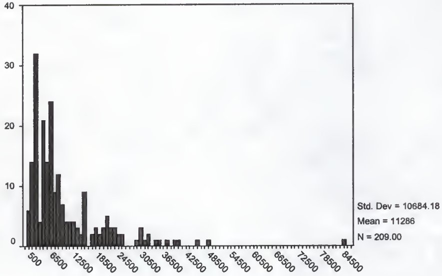
We have shown that there is indeed an auction mechanism that can be designed to adapt to the needs inherent to the complex environment of negotiations. We have summarized our conclusions concerning the performance characteristic of such a mechanism. This chapter presented various limitations to the research that suggest avenues for further research. The Incompletely Specified Combinatorial Auction is an auction model in its infancy, thus the mechanism poses great opportunities for further development and refinement.

APPENDIX A

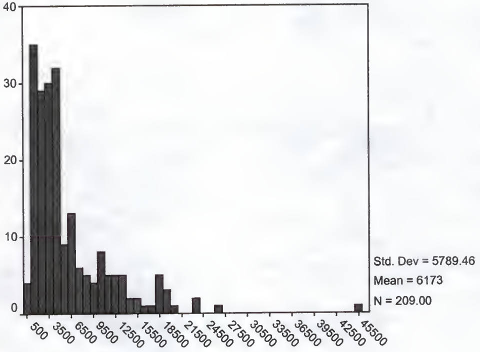
DISTRIBUTION OF DEMOGRAPHIC REQUIRMENTS BY CATEGORY



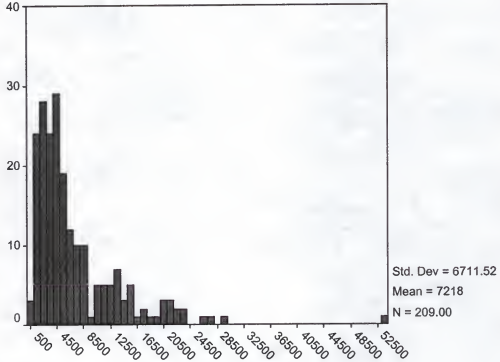
Women 18-49



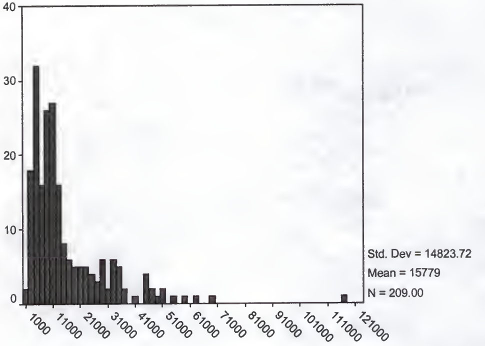
Women 25-54



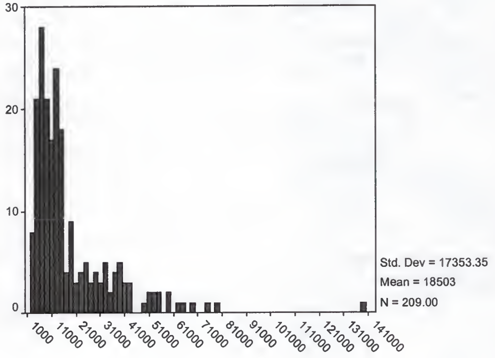
Men 18-49



Men 25-54



Adults 18-49

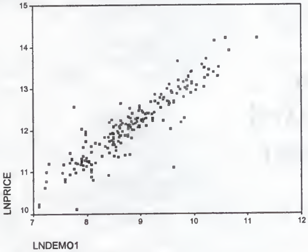


Adults 25-54

APPENDIX B

REGRESSION OF PRICE AND DEMOGRAPHICS

Demographic 1 Women 18-49



Regression Women 18-49 and Price

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.914	.836	.835	.3204

a Predictors: (Constant), LNDEMO1

ANOVA

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	108.425	1	108.425	1056.437	.000
	Residual	21.245	207	.103		
	Total	129.670	208			

a Predictors: (Constant), LNDEMO1

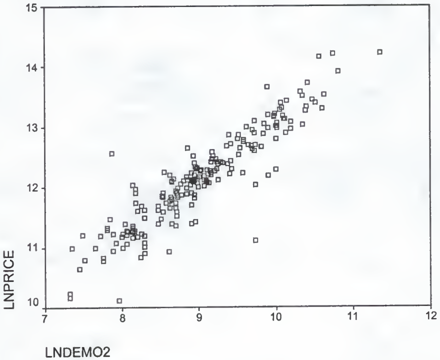
b Dependent Variable: LNPRICE

Coefficients

Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.
1	(Constant)	4.258	.243		17.505	.000
	LNDEMO1	.891	.027	.914	32.503	.000

a Dependent Variable: LNPRICE

Demographic 2 Women 25-54



Regression Women 25-54 and Price

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.915	.837	.837	.3192

a Predictors: (Constant), LNDEMO2

ANOVA

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	108.575	1	108.575	1065.432	.000
	Residual	21.095	207	.102		
	Total	129.670	208			

a Predictors: (Constant), LNDEMO2
b Dependent Variable: LNPRICE

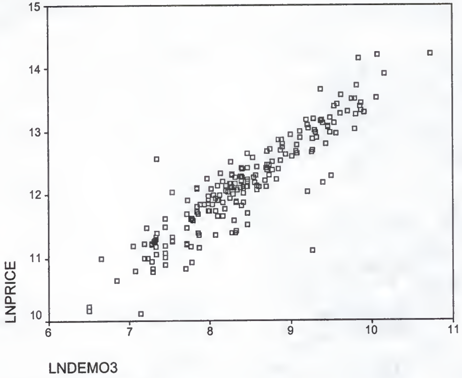
Coefficients

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	4.049	.249		16.285	.000
	LNDEMO2	.898	.028	.915	32.641	.000

a Dependent Variable: LNPRICE

Demographic 3 Men 18-49

Demographic 3 Men 18-49



Regression Men 18-49 and Price

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.914	.835	.834	.3215

a Predictors: (Constant), LNDEMO3

ANOVA

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	108.278	1	108.278	1047.752	.000
	Residual	21.392	207	.103		
	Total	129.670	208			

a Predictors: (Constant), LNDEMO3

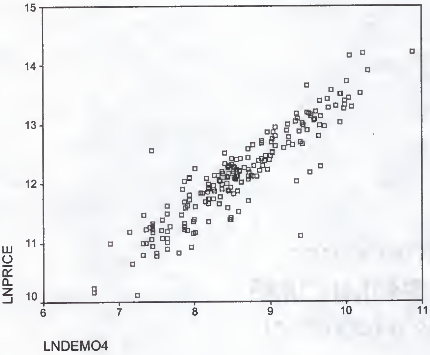
b Dependent Variable: LNPRICE

Coefficients

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	4.629	.233		19.883	.000
	LNDEMO3	.894	.028	.914	32.369	.000

a Dependent Variable: LNPRICE

Demographic 4 Men 25-54



Regression Men 25-54 and Price

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.913	.834	.833	.3227

a Predictors: (Constant), LNDEMO4

ANOVA

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	108.115	1	108.115	1038.267	.000
	Residual	21.555	207	.104		
	Total	129.670	208			

a Predictors: (Constant), LNDEMO4

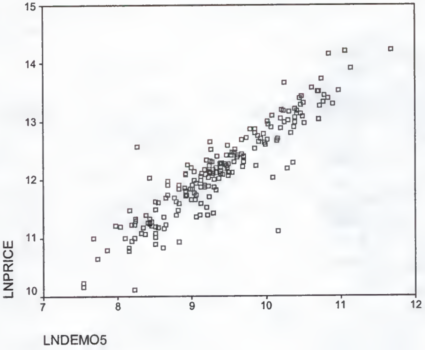
b Dependent Variable: LNPRICE

Coefficients

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	4.414	.241		18.353	.000
	LNDEMO4	.902	.028	.913	32.222	.000

a Dependent Variable: LNPRICE

Demographic 5 Adults 18-49



Regression Adults 18-49

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.919	.844	.843	.3127

a Predictors: (Constant), LNDEMO5

ANOVA

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	109.425	1	109.425	1118.842	.000
	Residual	20.245	207	9.780E-02		
	Total	129.670	208			

a Predictors: (Constant), LNDEMO5

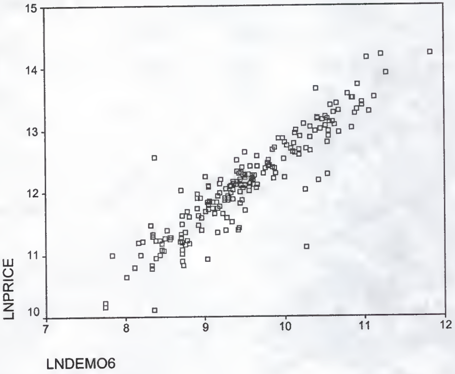
b Dependent Variable: LNPRICE

Coefficients

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	3.709	.253		14.674	.000
	LNDEMO5	.902	.027	.919	33.449	.000

a Dependent Variable: LNPRICE

Demographic 6 Adults 25-54



Regression Adults 25-54

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.920	.846	.845	.3108

a Predictors: (Constant), LNDEMO6

ANOVA

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	109.675	1	109.675	1135.407	.000
	Residual	19.995	207	9.660E-02		
	Total	129.670	208			

a Predictors: (Constant), LNDEMO6

b Dependent Variable: LNPRICE

Coefficients

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	3.468	.258		13.444	.000
	LNDEMO6	.912	.027	.920	33.696	.000

a Dependent Variable: LNPRICE

APPENDIX C
EXPERIMENTAL RESULTS

Table C1 Activity Rule – 5% Increment – 5 & 15 Minute – Set 1

Experiment - 5% Increments - Various Round Lengths							
Minutes Per Round	**Bidder Set**	**Minimum Bid Increment**	**Closing Rule**	**Minutes Per Round**	**Bidder Set**	**Minimum Bid Increment**	**Closing Rule**
5	Generic	5%	Activity	15	Generic	5%	Activity
Round	**# Bids**	**Revenue**	**Optimality**	**Round**	**# Bids**	**Revenue**	**Optimality**
1	255	18546.60	87.9986	1	255	18546.60	87.9986
2	278	19558.90	88.1961	2	278	20479.10	88.8792
3	254	20474.80	89.1492	3	253	21575.60	89.3750
4	240	21527.20	90.1414	4	237	22217.10	90.1766
5	224	23146.70	91.0156	5	220	22881.80	90.6210
6	203	23506.00	91.5384	***6***	196	23487.20	92.4770
7	195	23841.30	93.1144	7	186	23815.00	92.9266
8	168	23890.60	93.6205	8	169	24023.90	93.6125
9	160	24387.60	93.4635	9	148	24154.20	94.4255
10	152	24387.60	93.4635	10	145	24522.30	93.8447
11	146	24387.60	93.4635	11	142	24522.30	93.8447
12	141	24387.60	93.4635	12	137	24522.30	93.8447
13	138	24391.70	93.4445	13	135	24522.30	93.8447
14	136	24391.70	93.4445	14	134	24522.30	93.8447
15	143	24391.70	93.4445	15	143	24578.50	93.6227
16	140	24391.70	93.4445	16	141	24578.50	93.6227
17	139	24391.70	93.4445	17	141	24578.50	93.6227
18	134	24391.70	93.4445	18	137	24578.50	93.6227
19	133	24391.70	93.4445	19	135	24578.50	93.6227
20	132	24391.70	93.4445	20	134	24578.50	93.6227
				21	133	24578.50	93.6227
Efficiency:	94.7058	Gap:	11.7144	Efficiency:	96.065	Gap:	4.8607

Table C2 Activity Rule – 5% Increment – 5 & 15 Minute – Set 2

Experiment - 5% Increments - Various Round Lengths							
Minutes Per Round	**Bidder Set**	**Minimum Bid Increment**	**Closing Rule**	**Minutes Per Round**	**Bidder Set**	**Minimum Bid Increment**	**Closing Rule**
5	Generic	5%	Activity	15	Generic	5%	Activity
Round	**# Bids**	**Revenue**	**Optimality**	**Round**	**# Bids**	**Revenue**	**Optimality**
Set 2							
1	248	19142.10	88.2187	1	248	19204.50	88.1924
2	269	20857.30	89.7142	2	269	20937.30	89.7495
3	247	22077.60	90.9819	3	249	22837.90	90.9757
4	235	22835.10	91.2939	4	232	22837.90	90.9757
5	217	23222.40	92.1958	5	206	22930.70	91.0544
6	190	23457.80	92.1380	***6***	186	23203.60	91.9155
7	176	23938.30	92.8715	7	176	23535.60	92.3030
8	159	23938.30	92.8715	8	153	23659.60	93.5633
9	144	23973.00	93.0584	9	144	23979.70	92.9056
10	145	24153.10	93.1164	10	144	24138.00	92.6800
11	138	24153.10	93.1164	11	141	24138.00	92.6800
12	136	24155.70	93.1354	12	137	24138.00	92.6800
13	134	24155.70	93.1354	13	135	24138.00	92.6800
14	134	24155.70	93.1354	14	135	24138.00	92.6800
15	141	24155.70	93.1354	15	142	24138.00	92.6800
16	140	24155.70	93.1354	16	139	24194.80	92.9077
17	136	24409.40	93.2974	17	136	24320.80	93.0409
18	134	24409.40	93.2974	18	135	24320.80	93.0409
19	133	24409.40	93.2974	19	133	24320.80	93.0409
20	132	24409.40	93.2974	20	129	24320.80	93.0409
21	131	24409.40	93.2974	21	129	24334.10	93.3839
22	131	24409.40	93.2974	22	129	24334.10	93.3839
23	130	24409.40	93.2974	23	129	24334.10	93.3839
				24	128	24334.10	93.3839
				25	128	24334.10	93.3839
				26	127	24334.10	93.3839
Efficiency:	96.4054	Gap:	5.4141	Efficiency:	95.5788	Gap:	5.2129

Table C3 Activity Rule – 5% Increment – 30 & 60 Minute – Set 1

Experiment - 5% Increments - Various Round Lengths							
Minutes Per Round	**Bidder Set**	**Minimum Bid Increment**	**Closing Rule**	**Minutes Per Round**	**Bidder Set**	**Minimum Bid Increment**	**Closing Rule**
30	Generic	5%	Activity	60	Generic	5%	Activity
Round	**# Bids**	**Revenue**	**Optimality**	**Round**	**# Bids**	**Revenue**	**Optimality**
1	255	18921.50	86.7711	1	255	18921.50	86.7711
2	278	20450.30	88.7718	2	278	20450.30	88.7718
3	256	20944.60	89.9528	3	256	20944.60	89.9528
4	234	22720.60	90.5945	4	234	22816.30	90.8370
5	217	23041.10	91.5958	5	216	23112.00	91.6281
6	187	23361.50	92.3777	***6***	187	23315.90	92.2291
7	183	23816.30	93.5409	7	183	23759.80	93.8397
8	169	23816.30	93.5409	8	166	23961.10	93.4099
9	154	24175.20	93.7210	9	154	23972.40	93.4551
10	145	24279.50	93.7379	10	147	24165.60	93.4205
11	141	24279.50	93.7379	11	141	24184.10	93.6485
12	137	24279.50	93.7379	12	137	24184.10	93.6485
13	134	24279.50	93.7379	13	135	24184.10	93.6485
14	133	24279.50	93.7379	14	135	24283.00	94.0178
15	142	24279.50	93.7379	15	144	24283.00	94.0178
16	138	24279.50	93.7379	16	141	24341.90	94.1552
17	136	24279.50	93.7379	17	138	24341.90	94.1552
18	134	24279.50	93.7379	18	136	24341.90	94.1552
19	132	24279.50	93.7379	19	134	24392.30	94.8055
20	131	24279.50	93.7379	20	130	24473.50	94.6952
				21	130	24473.50	94.6952
				22	126	24473.50	94.6952
				23	125	24473.50	94.6952
Efficiency:	94.879	Gap:	7.5142	Efficiency:	98.6405	Gap:	1.8511

Table C4 Activity Rule – 5% Increment – 30 & 60 Minute – Set 2

Experiment - 5% Increments - Various Round Lengths							
Minutes Per Round	**Bidder Set**	**Minimum Bid Increment**	**Closing Rule**	**Minutes Per Round**	**Bidder Set**	**Minimum Bid Increment**	**Closing Rule**
30	Generic	5%	Activity	60	Generic	5%	Activity
Round	**# Bids**	**Revenue**	**Optimality**	**Round**	**# Bids**	**Revenue**	**Optimality**
Set 2							
1	248	19204.50	88.1924	1	248	19204.50	88.1924
2	269	20937.30	89.7495	2	269	20937.30	89.7495
3	249	22837.90	90.9757	3	249	22837.90	90.9757
4	232	23106.70	90.8965	4	232	23106.70	90.8965
5	217	23384.20	92.5969	***5***	217	23386.40	92.5065
6	197	23640.50	92.5408	6	196	23540.10	92.8275
7	175	23853.80	93.0955	7	176	23918.50	92.4601
8	162	24117.60	92.8127	8	155	24145.60	92.7926
9	148	24118.50	93.4416	9	147	24145.60	92.7926
10	147	24192.50	93.2168	10	143	24356.90	92.5507
11	144	24214.90	93.3793	11	138	24356.90	92.5507
12	142	24306.60	93.5021	12	134	24356.90	92.5507
13	142	24486.60	93.9484	13	134	24356.90	92.5507
14	141	24621.10	93.8396	14	134	24356.90	92.5507
15	146	24633.80	93.7369	15	143	24356.90	92.5507
16	143	24656.80	93.6082	16	139	24356.90	92.5507
17	138	24565.80	93.6082	17	135	24356.90	92.5507
18	136	24565.80	93.6082	18	133	24356.90	92.5507
19	134	24565.80	93.6082	19	132	24381.70	92.9378
20	133	24565.80	93.6082	20	129	24381.70	92.9378
Efficiency:	95.2037	Gap:	7.1454	Efficiency:	94.0463	Gap:	7.7778

Table C5 Activity Rule – 10% Increment – 5 & 15 Minute – Set 1

Experiment -10% Increments-Various Round Lengths							
Minutes Per Round	**Bidder Set**	**Minimum Bid Increment**	**Closing Rule**	**Minutes Per Round**	**Bidder Set**	**Minimum Bid Increment**	**Closing Rule**
5	Generic	10%	Activity	15	Generic	10%	Activity
Round	**# Bids**	**Revenue**	**Optimality**	**Round**	**# Bids**	**Revenue**	**Optimality**
1	255	18546.60	87.9986	1	255	18546.60	87.9986
2	272	20931.20	89.4097	2	272	20931.20	89.4097
3	246	23097.20	92.5025	3	246	23097.20	92.5025
4	212	23539.20	94.2181	***4***	212	23636.30	93.7651
5	185	23781.80	94.0913	5	184	24207.30	94.5288
6	162	24802.40	95.7664	6	156	24705.70	95.2158
7	156	24914.30	95.1333	7	150	25052.10	95.2880
8	149	24962.90	95.4638	8	145	25052.10	95.2880
9	141	24962.90	95.4638	9	138	25052.10	95.2880
10	143	25004.70	95.2536	10	137	25052.10	95.2880
11	139	25004.70	95.2536	11	136	25052.10	95.2880
12	137	25090.90	95.4766	12	133	25052.10	95.2880
13	136	25170.40	95.5566	13	133	25052.10	95.2880
14	136	25170.40	95.5566	14	132	25052.10	95.2880
15	145	25170.40	95.5566	15	140	25052.10	95.2880
16	139	25170.40	95.5566	16	136	25052.10	95.2880
17	138	25196.70	95.5597	17	135	25052.10	95.2880
18	136	25264.10	95.5487	18	133	25052.10	95.2880
19	135	25264.10	95.5487	19	131	25052.10	95.2880
20	135	25264.10	95.5487	20	131	25052.10	95.2880
21	134	25264.10	95.5487	21	130	25052.10	95.2880
22	134	25264.10	95.5487				
23	134	25264.10	95.5487				
24	133	25264.10	95.5487				
Efficiency:	97.4008	Gap:	5.8607	Efficiency:	96.5083	Gap:	7.2824

Table C6 Activity Rule – 10% Increment – 5 & 15 Minute – Set 2

Experiment -10% Increments-Various Round Lengths							
Minutes Per Round	**Bidder Set**	**Minimum Bid Increment**	**Closing Rule**	**Minutes Per Round**	**Bidder Set**	**Minimum Bid Increment**	**Closing Rule**
5	Generic	10%	Activity	15	Generic	10%	Activity
Round	**# Bids**	**Revenue**	**Optimality**	**Round**	**# Bids**	**Revenue**	**Optimality**
Set 2							
1	248	19142.10	88.2187	1	248	19204.50	88.1924
2	260	21074.80	90.4370	2	261	21891.30	90.8288
3	234	22886.20	92.4315	3	231	22686.70	91.5543
4	203	23532.20	93.2763	***4***	202	23498.30	93.3097
5	176	23855.70	93.7636	5	183	23986.90	93.4585
6	155	24152.30	93.9759	6	153	24027.10	93.8715
7	152	24372.00	93.7524	7	150	24324.80	93.9720
8	142	24372.00	93.7524	8	141	24324.80	93.9720
9	140	24372.00	93.7524	9	136	24324.80	93.9720
10	141	24372.00	93.7524	10	140	24370.70	93.9918
11	137	24461.60	94.0362	11	138	24421.20	94.3690
12	135	24461.60	94.0362	12	136	24588.60	94.3596
13	135	24641.60	94.0773	13	133	24588.60	94.3596
14	135	24641.60	94.0773	14	131	24588.60	94.3596
15	141	24641.60	94.0773	15	139	24588.60	94.3596
16	139	24641.60	94.0773	16	138	24588.60	94.3596
17	135	24641.60	94.0773	17	136	24588.60	94.3596
18	133	24641.60	94.0773	18	132	24588.60	94.3596
19	133	24641.60	94.0773	19	131	24588.60	94.3596
20	132	24641.60	94.0773	20	131	24588.60	94.3596
				21	130	24588.60	94.3596
Efficiency:	95.2012	Gap:	8.8182	Efficiency:	95.9128	Gap:	11.1256

Table C7 Activity Rule – 10% Increment – 30 & 60 Minute – Set 1

Experiment -10% Increments-Various Round Lengths							
Minutes Per Round	**Bidder Set**	**Minimum Bid Increment**	**Closing Rule**	**Minutes Per Round**	**Bidder Set**	**Minimum Bid Increment**	**Closing Rule**
30	Generic	10%	Activity	60	Generic	10%	Activity
Round	**# Bids**	**Revenue**	**Optimality**	**Round**	**# Bids**	**Revenue**	**Optimality**
1	255	18921.50	86.7711	1	255	18921.50	86.7711
2	273	20866.00	89.8837	2	273	20866.00	89.8837
3	251	22671.10	91.4325	3	251	23013.40	92.2690
4	227	23693.90	93.7658	***4***	218	23776.50	94.2215
5	195	24300.50	94.4896	5	182	24244.90	94.6758
6	157	24479.90	95.3027	6	155	24616.90	95.3836
7	154	24982.90	95.2842	7	150	24922.70	95.4581
8	142	24982.90	95.2842	8	141	25057.20	95.5444
9	135	24982.90	95.2842	9	133	25057.20	95.5444
10	135	24982.90	95.2842	10	134	25057.20	95.5444
11	135	24982.90	95.2842	11	133	25057.20	95.5444
12	131	24982.90	95.2842	12	130	25057.20	95.5444
13	130	25047.50	95.5340	13	129	25057.20	95.5444
14	129	25047.50	95.5340	14	128	25069.00	95.6060
15	138	25047.50	95.5340	15	137	25069.00	95.6060
16	134	25047.50	95.5340	16	134	25069.00	95.6060
17	132	25047.50	95.5340	17	131	25069.00	95.6060
18	128	25047.50	95.5340	18	127	25069.00	95.6060
19	127	25047.50	95.5340	19	126	25069.00	95.6060
Efficiency:	96.0483	Gap:	5.6402	Efficiency:	96.7611	Gap:	9.6151

Table C8 Activity Rule – 10% Increment – 30 & 60 Minute – Set 2

Experiment -10% Increments-Various Round Lengths							
Minutes Per Round	**Bidder Set**	**Minimum Bid Increment**	**Closing Rule**	**Minutes Per Round**	**Bidder Set**	**Minimum Bid Increment**	**Closing Rule**
30	Generic	10%	Activity	60	Generic	10%	Activity
Round	**# Bids**	**Revenue**	**Optimality**	**Round**	**# Bids**	**Revenue**	**Optimality**
Set 2							
1	248	19204.50	88.1924	1	248	19204.50	88.1924
2	261	21891.30	90.8288	2	261	21891.30	90.8288
3	231	22686.70	91.5543	3	231	22686.70	91.5543
4	202	23498.30	93.3097	***4***	202	23432.90	92.8737
5	183	23836.10	93.2136	5	182	23833.20	93.1977
6	154	24124.30	94.1025	6	159	24205.40	94.0264
7	149	24282.50	93.9223	7	150	24315.10	93.7581
8	142	24342.90	93.6358	8	143	24321.50	94.2648
9	139	24354.20	94.1615	9	138	24451.00	94.1833
10	141	24465.70	94.1486	10	140	24451.00	94.1833
11	139	24623.70	94.2581	11	138	24451.00	94.1833
12	139	24623.70	94.2581	12	135	24473.90	94.4432
13	135	24623.70	94.2581	13	135	24473.90	94.4432
14	134	24650.80	94.3572	14	135	24495.60	94.9036
15	143	24650.80	94.3572	15	142	24742.50	94.6947
16	141	24650.80	94.3572	16	140	24742.50	94.6947
17	136	24650.80	94.3572	17	139	24742.50	94.6947
18	134	24650.80	94.3572	18	136	24742.50	94.6947
19	132	24650.80	94.3572	19	133	24742.50	94.6947
20	132	24650.80	94.3572	20	133	24742.50	94.6947
21	131	24650.80	94.3572	21	132	24742.50	94.6947
Efficiency:	94.7756	Gap:	8.3092	Efficiency:	96.1138	Gap:	12.4754

Table C9 Activity Rule – 15% Increment – 5 & 15 Minute – Set 1

Experiment -15% Increments-Various Round Lengths							
Minutes Per Round	**Bidder Set**	**Minimum Bid Increment**	**Closing Rule**	**Minutes Per Round**	**Bidder Set**	**Minimum Bid Increment**	**Closing Rule**
5	Generic	15%	Activity	15	Generic	15%	Activity
Round	**# Bids**	**Revenue**	**Optimality**	**Round**	**# Bids**	**Revenue**	**Optimality**
1	255	18546.60	87.9986	1	255	18546.60	87.9986
2	258	21922.70	91.1307	2	258	22079.90	91.1946
3	229	23377.90	93.3870	***3***	224	23284.90	93.7843
4	197	24256.40	95.4258	4	191	24209.30	94.9298
5	173	24652.60	95.4636	5	168	24731.50	95.2879
6	152	24790.20	95.8046	6	154	24731.50	95.2879
7	146	24828.60	95.6090	7	142	24731.50	95.2879
8	136	24869.20	95.7468	8	135	24731.50	95.2879
9	134	24869.20	95.7468	9	132	24731.50	95.2879
10	136	24869.20	95.7468	10	132	24731.50	95.2879
11	134	24869.20	95.7468	11	131	24736.50	95.2888
12	131	24877.50	95.9355	12	128	24742.00	95.3353
13	130	24921.50	96.0266	13	127	24818.50	95.4169
14	127	24979.40	96.2838	14	125	24818.50	95.4169
15	133	24979.40	96.2838	15	132	24818.50	95.4169
16	129	24979.40	96.2838	16	129	24818.50	95.4169
17	126	24979.40	96.2838	17	124	24818.50	95.4169
18	124	24979.40	96.2838	18	123	24818.50	95.4169
19	123	24979.40	96.2838	19	122	24818.50	95.4169
Efficiency:	96.9639	Gap:	4.57951	Efficiency:	96.7413	Gap:	13.4971

Table C10 Activity Rule – 15% Increment – 5 & 15 Minute – Set 2

Experiment -15% Increments-Various Round Lengths							
Minutes Per Round	Bidder Set	Minimum Bid Increment	Closing Rule	Minutes Per Round	Bidder Set	Minimum Bid Increment	Closing Rule
5	Generic	15%	Activity	15	Generic	15%	Activity
Round	# Bids	Revenue	Optimality	Round	# Bids	Revenue	Optimality
Set 2							
1	248	19142.10	88.2187	1	248	19204.50	88.1924
2	249	20734.40	91.0614	2	249	21724.70	90.6287
3	216	23298.00	92.9948	***3***	217	23395.00	93.2119
4	196	23432.40	93.9796	4	198	24145.60	94.3555
5	175	23432.40	93.9796	5	170	24322.50	94.2412
6	159	23593.50	94.7172	6	149	24449.30	94.4447
7	155	23593.50	94.7172	7	146	24481.90	94.5034
8	149	23593.50	94.7172	8	138	24575.10	94.3865
9	148	23593.50	94.7172	9	136	24575.10	94.3865
10	151	23593.50	94.7172	10	140	24575.10	94.3865
11	147	23593.50	94.7172	11	137	24695.10	94.6493
12	147	23593.50	94.7172	12	135	24760.40	95.0584
13	146	23593.50	94.7172	13	130	24817.50	94.9181
14	144	23593.50	94.7172	14	129	24817.50	94.9181
15	153	23593.50	94.7172	15	138	24849.00	94.6865
16	149	23593.50	94.7172	16	133	24849.00	94.6865
17	146	23593.50	94.7172	17	133	24849.00	94.6865
18	144	23593.50	94.7172	18	130	24849.00	94.6865
19	144	23593.50	94.7172	19	129	24849.00	94.6865
20	144	23593.50	94.7172				
21	143	23593.50	94.7172				
Efficiency:	97.3315	Gap:	3.2963	Efficiency:	94.9535	Gap:	5.8667

Table C11 Activity Rule – 15% Increment – 30 & 60 Minute – Set 1

Experiment -15% Increments-Various Round Lengths							
Minutes Per Round	**Bidder Set**	**Minimum Bid Increment**	**Closing Rule**	**Minutes Per Round**	**Bidder Set**	**Minimum Bid Increment**	**Closing Rule**
30	Generic	15%	Activity	60	Generic	15%	Activity
Round	**# Bids**	**Revenue**	**Optimality**	**Round**	**# Bids**	**Revenue**	**Optimality**
1	248	19142.10	88.2187	1	255	18921.50	86.7711
2	258	22354.80	90.6081	2	258	22354.80	90.6081
3	223	23395.10	93.6812	***3***	223	23395.10	93.6812
4	197	24256.10	95.2656	4	197	24319.30	95.3356
5	167	24738.30	95.7055	5	165	24775.80	95.3783
6	149	24859.70	95.7925	6	148	24901.60	95.3821
7	146	24873.10	95.6688	7	145	24901.60	95.3821
8	140	25025.50	95.4377	8	138	24940.70	95.3822
9	137	25025.50	95.4377	9	136	24940.70	95.3822
10	140	25025.50	95.4377	10	139	24940.70	95.3822
11	139	25025.50	95.4377	11	137	24985.10	95.7504
12	136	25056.90	95.4432	12	136	25010.20	95.6194
13	135	25056.90	95.4432	13	133	25010.20	95.6194
14	134	25056.90	95.4432	14	133	25010.20	95.6194
15	142	25056.90	95.4432	15	139	25010.20	95.6194
16	138	25056.90	95.4432	16	136	25010.20	95.6194
17	136	25056.90	95.4432	17	131	25010.20	95.6194
18	134	25056.90	95.4432	18	131	25010.20	95.6194
19	133	25056.90	95.4432	19	131	25010.20	95.6194
				20	130	25010.20	95.6194
Efficiency:	95.757	Gap:	8.2197	Efficiency:	96.4832	Gap:	12.4506

Table C12 Activity Rule – 15% Increment – 30 & 60 Minute – Set 2

Experiment -15% Increments-Various Round Lengths							
Minutes Per Round	**Bidder Set**	**Minimum Bid Increment**	**Closing Rule**	**Minutes Per Round**	**Bidder Set**	**Minimum Bid Increment**	**Closing Rule**
30	Generic	15%	Activity	60	Generic	15%	Activity
Round	**# Bids**	**Revenue**	**Optimality**	**Round**	**# Bids**	**Revenue**	**Optimality**
Set 2							
1	248	19204.50	88.1924	1	248	19204.50	88.1924
2	249	21724.70	90.6287	2	249	21724.70	90.6287
3	217	23237.90	93.2119	***3***	217	23395.00	93.2119
4	197	24122.40	94.5231	4	198	24145.60	94.3555
5	176	24503.50	94.0921	5	170	24322.50	94.2412
6	147	24517.90	94.1268	6	149	24449.30	94.4447
7	147	24540.20	94.1814	7	146	24481.90	94.5034
8	140	24604.50	94.0115	8	138	24575.10	94.3865
9	138	24825.30	94.0886	9	136	24575.10	94.3865
10	140	24831.00	94.1542	10	140	24575.10	94.3865
11	139	24919.30	94.1737	11	137	24695.10	94.6493
12	138	24954.00	94.1574	12	135	24760.40	95.0584
13	138	25001.70	94.1918	13	130	24817.50	94.9181
14	138	25001.70	94.1918	14	129	24817.50	94.9181
15	146	25001.70	94.1918	15	138	24849.00	94.6865
16	144	25001.70	94.1918	16	133	24849.00	94.6865
17	141	25001.70	94.1918	17	133	24849.00	94.6865
18	137	25001.70	94.1918	18	130	24849.00	94.6865
19	137	25001.70	94.1918	19	129	24849.00	94.6865
20	136	25001.70	94.1918				
Efficiency:	94.7003	Gap:	7.9444	Efficiency:	94.9535	Gap:	5.8667

Table C13 Minimum Revenue Rule – 5% Increment – 5 & 15 Minute

Experiment - 5% Increments - Various Round Lengths							
Minutes Per Round	**Bidder Set**	**Minimum Bid Increment**	**Closing Rule**	**Minutes Per Round**	**Bidder Set**	**Minimum Bid Increment**	**Closing Rule**
5	Generic	5%	MinRev	15	Generic	5%	MinRev
Round	**# Bids**	**Revenue**	**Optimality**	**Round**	**# Bids**	**Revenue**	**Optimality**
1	255	18546.60	87.9986	1	255	18546.60	87.9986
2	278	19558.90	88.1961	2	278	20479.10	88.8792
3	254	20474.80	89.1492	3	253	21575.60	89.3750
4	240	21527.20	90.1414	4	237	22217.10	90.1766
5	224	23146.70	91.0156	5	220	22881.80	90.6210
				6	196	23487.20	92.4770
Efficiency:	89.4526	Gap:	21.2059	Efficiency:	90.8648	20.3426	20.8953
Set 2							
1	248	19142.10	88.2187	1	248	19204.50	88.1924
2	269	20857.30	89.7142	2	269	20937.30	89.7495
3	247	22084.50	90.9464	3	249	22837.90	90.9757
4	235	23035.30	91.4878	4	232	22837.90	90.9757
5	217	23035.30	91.4878	5	206	22930.70	91.0544
6	185	23498.80	92.3043	***6***	186	23203.60	91.9155
Efficiency:	91.6006	Gap:	20.1352	Efficiency:	90.6344	Gap:	21.2625

Table C14 Minimum Revenue Rule – 10% Increment – 5 & 15 Minute

Experiment -10% Increments-Various Round Lengths							
Minutes Per Round	**Bidder Set**	**Minimum Bid Increment**	**Closing Rule**	**Minutes Per Round**	**Bidder Set**	**Minimum Bid Increment**	**Closing Rule**
5	Generic	10%	MinRev	15	Generic	10%	MinRev
Round	**# Bids**	**Revenue**	**Optimality**	**Round**	**# Bids**	**Revenue**	**Optimality**
1	255	18546.60	87.9986	1	255	18546.60	87.9986
2	272	20931.20	89.4097	2	272	20931.20	89.4097
3	246	23097.20	92.5025	3	246	23097.20	92.5025
4	212	23538.10	93.9426	***4***	212	23617.70	94.0618
Efficiency:	91.5521	Gap:	22.1833	Efficiency:	91.8615	Gap:	21.8349
Set 2							
1	248	19142.10	88.2187	1	248	19204.50	88.1924
2	260	21074.80	90.4370	2	261	21891.30	90.8288
3	234	22890.80	92.4501	3	231	22686.70	91.5543
4	204	23586.20	93.4855	***4***	202	23498.30	93.3097
Efficiency:	91.503	Gap:	21.5993	Efficiency:	90.3863	Gap:	20.8866

Table C15 Minimum Revenue Rule – 15% Increment – 5 & 15 Minute

Experiment -15% Increments-Various Round Lengths							
Minutes Per Round	**Bidder Set**	**Minimum Bid Increment**	**Closing Rule**	**Minutes Per Round**	**Bidder Set**	**Minimum Bid Increment**	**Closing Rule**
5	Generic	15%	MinRev	15	Generic	15%	MinRev
Round	**# Bids**	**Revenue**	**Optimality**	**Round**	**# Bids**	**Revenue**	**Optimality**
1	255	18546.60	87.9986	1	255	18546.60	87.9986
2	258	21922.70	91.1307	2	258	22079.90	91.1946
3	229	23377.90	93.3870	***3***	224	23284.90	93.7843
Efficiency:	98.002	Gap:	21.6325	Efficiency:	91.2745	Gap:	21.8929
Set 2							
1	248	19142.10	88.2187	1	248	19204.50	88.1924
2	249	20734.40	91.0614	2	249	21724.70	90.6287
3	216	23298.00	92.9948	***3***	217	23395.00	93.2119
Efficiency:	97.46	Gap:	22.1324	Efficiency:	91.8082	Gap:	20.7417

Table C16 Maximum Round=10 Rule – 5% Increment – 5 & 15 Minute

Experiment - 5% Increments - Various Round Lengths							
Minutes Per Round	**Bidder Set**	**Minimum Bid Increment**	**Closing Rule**	**Minutes Per Round**	**Bidder Set**	**Minimum Bid Increment**	**Closing Rule**
5	Generic	5%	Round=10	15	Generic	5%	Round=10
Round	**# Bids**	**Revenue**	**Optimality**	**Round**	**# Bids**	**Revenue**	**Optimality**
1	255	18546.60	87.9986	1	255	18546.60	87.9986
2	278	19558.90	88.1961	2	278	20479.10	88.8792
3	254	20474.80	89.1492	3	253	21575.60	89.3750
4	240	21527.20	90.1414	4	237	22217.10	90.1766
5	224	23146.70	91.0156	5	220	22881.80	90.6210
6	203	23506.00	91.5384	***6***	196	23487.20	92.4770
7	195	23841.30	93.1144	7	186	23815.00	92.9266
8	168	23890.60	93.6205	8	169	24023.90	93.6125
9	160	24387.60	93.4635	9	148	24154.20	94.4255
10	152	24387.60	93.4635	10	145	24522.30	93.8447
Efficiency:	93.9347	Gap:	16.3186	Efficiency:	94.8273	Gap:	15.5720
Set 2							
1	248	19142.10	88.2187	1	248	19204.50	88.1924
2	269	20857.30	89.7142	2	269	20937.30	89.7495
3	247	22084.50	90.9464	3	249	22837.90	90.9757
4	235	23035.30	91.4878	4	232	22837.90	90.9757
5	235	23035.30	91.4878	5	206	22930.70	91.0544
6	194	23851.30	92.1951	***6***	186	23203.60	91.9155
7	184	23998.40	93.2668	7	176	23535.60	92.3030
8	168	24287.20	93.9622	8	153	23659.60	93.5633
9	162	24287.20	93.9622	9	144	23979.70	92.9056
10	159	24287.20	93.9622	10	144	24138.00	92.6800
Efficiency:	93.1122	Gap:	16.4597	Efficiency:	92.5738	Gap:	17.2806

Table C17 Maximum Round=10 Rule – 10% Increment – 5 & 15 Minute

Experiment -10% Increments-Various Round Lengths							
Minutes Per Round	Bidder Set	Minimum Bid Increment	Closing Rule	Minutes Per Round	Bidder Set	Minimum Bid Increment	Closing Rule
5	Generic	10%	Round=10	15	Generic	10%	Round=10
Round	# Bids	Revenue	Optimality	Round	# Bids	Revenue	Optimality
1	255	18546.60	87.9986	1	255	18546.60	87.9986
2	272	20931.20	89.4097	2	272	20931.20	89.4097
3	246	23097.20	92.5025	3	246	23097.20	92.5025
4	212	23641.70	94.3282	***4***	212	23543.50	93.6646
5	187	23767.80	94.0879	5	183	24250.90	94.7524
6	162	24594.70	95.7148	6	154	24590.10	95.2736
7	156	24919.10	95.3553	7	147	24907.30	94.9794
8	147	24919.10	95.3553	8	143	25015.60	95.4790
9	139	24919.10	95.3553	9	137	25015.60	95.4790
10	139	24919.10	95.3553	10	138	25143.20	95.4414
Efficiency:	95.052	Gap:	10.9086	Efficiency:	95.7361	Gap:	11.9127
Set 2							
1	248	19142.10	88.2187	1	248	19204.50	88.1924
2	260	21074.80	90.4370	2	261	21891.30	90.8288
3	234	22886.20	92.4315	3	231	22686.70	91.5543
4	203	23508.40	93.2528	***4***	202	23498.30	93.3097
5	179	23897.80	93.9488	5	183	23986.90	93.4585
6	158	24308.50	94.1326	6	153	24027.10	93.8715
7	148	24392.30	94.2361	7	150	24324.80	93.9720
8	146	24392.30	94.2361	8	141	24324.80	93.9720
9	141	24467.90	94.3917	9	136	24324.80	93.9720
10	143	24467.90	94.3917	10	140	24370.70	93.9918
Efficiency:	94.0633	Gap:	16.7943	Efficiency:	94.1339	Gap:	14.2980

Table C18 Maximum Round=10 Rule – 15% Increment – 5 & 15 Minute

Experiment -15% Increments-Various Round Lengths							
Minutes Per Round	**Bidder Set**	**Minimum Bid Increment**	**Closing Rule**	**Minutes Per Round**	**Bidder Set**	**Minimum Bid Increment**	**Closing Rule**
5	Generic	15%	Round=10	15	Generic	15%	Round=10
Round	**# Bids**	**Revenue**	**Optimality**	**Round**	**# Bids**	**Revenue**	**Optimality**
1	255	18546.60	87.9986	1	255	18546.60	87.9986
2	258	21922.70	91.1307	2	258	22079.90	91.1946
3	229	23377.90	93.3870	***3***	224	23284.90	93.7843
4	197	24256.40	95.4258	4	191	24193.80	95.3229
5	173	24652.60	95.4636	5	170	24562.30	95.3437
6	152	24790.20	95.8046	6	149	24667.70	96.2084
7	146	24828.60	95.6090	7	140	25026.20	95.8695
8	136	24869.20	95.7468	8	137	25026.20	95.8695
9	134	24869.20	95.7468	9	136	25038.20	95.9120
10	136	24869.20	95.7468	10	136	25069.40	95.7616
Efficiency:	95.7839	Gap:	12.3736	Efficiency:	97.3613	Gap:	9.7123
Set 2							
1	248	19142.10	88.2187	1	248	19204.50	88.1924
2	249	20734.40	91.0614	2	249	21724.70	90.6287
3	216	23298.00	92.9948	***3***	217	23395.00	93.2119
4	196	23502.40	93.9535	4	198	24030.20	94.0111
5	176	23507.50	94.8602	5	173	24362.90	94.2297
6	161	23957.60	94.8943	6	154	24637.20	94.4737
7	162	24591.30	94.5547	7	149	24637.20	94.4737
8	156	24721.20	94.6581	8	144	24637.20	94.4737
9	150	24812.60	94.6767	9	140	24637.20	94.4737
10	153	24923.30	94.9689	10	143	24646.20	94.3543
Efficiency:	95.6039	Gap:	12.3772	Efficiency:	94.3855	Gap:	9.4156

Table C19 Unsold Inventory Activity Stopping Rule

Unsold Inventory																									
Show / Experiment	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	Total
Activity/MinRev																									
5Min5%		1		1		1			1																**4**
5Min10%							1				1		1				1								**4**
5Min15%																		1							**1**
15Min5%					1				1									1							**3**
15Min10%			1			1			1		1								1						**5**
15Min15%						1													1						**2**
30Min5%			1		1		1		1										1						**5**
30Min10%		1	1			1	1												1						**5**
30Min15%						1					1	1							1						**4**
60Min5%							1											1						1	**3**
60Min10%					1	1					1	1					1		1						**6**
60Min15%											1							1	1						**3**
Set 2																									
5Min5%						1					1														**2**
5Min10%											1			1				1	1						**4**
5Min15%						2						1					1		1						**5**
15Min5%			1		1				1		1														**4**
15Min10%									1		1			1											**3**
15Min15%									1		1		1						1						**4**
30Min5%											1								1						**2**
30Min10%											1														**1**
30Min15%						1												1	1						**3**
60Min5%				1			1					1		1											**4**
60Min10%											1														**1**
60Min15%									1		1		1						1						**4**

Table C20 Unsold Inventory Minimum Revenue Stopping Rule

Unsold Inventory																									
Show / Experiment	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	Total
MinRevenue																									
5Min5%					1								1												2
5Min10%				1			1					1	1					1							5
5Min15%												1	1												2
15Min5%																		1							1
15Min10%												1	1					1							3
15Min15%						1	1						1					1	1						5
Set 2																									
5Min5%				1	1								1						1						4
5Min10%							1					1		1				1							4
5Min15%					1														1						2
15Min5%				1	1																				2
15Min10%		1		1	1						1														4
15Min15%												1													1

Table C21 Unsold Inventory Maximum Round=10 Stopping Rule

Unsold Inventory																									
Show / Experiment	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	Total
MaxRound=10																									
5Min5%				1	1				1				1												4
5Min10%									1		1	1	1						1						5
5Min15%											1								1						2
15Min5%			1		1				1									1	1						5
15Min10%						1	1					1							1						4
15Min15%																	1	1	1						3
Set 2																									
5Min5%												1	1				1	1							4
5Min10%		1									1			1											3
5Min15%											1	1	1												3
15Min5%		1			1		1		1		1								1						6
15Min10%						1								1											2
15Min15%						1							1					1	1						4

Table C22 Winning Bidder Type Distribution - Activity Stopping Rule - Set 1

Winning Bidder Type Analysis							
Activity Stopping Rule (Set 1)							
Experiment		5Min5%C	15Min5%C	30Min5%C	60Min5%C	5Min10%C	15Min10%C
Evaluators		4	5	4	8	4	4
		3%	4%	3%	7%	3%	3%
Bid Adjustors	Min	28	26	29	29	27	29
		22%	20%	23%	24%	21%	23%
	Max	11	15	9	7	9	10
		9%	12%	7%	6%	7%	8%
All Adjustors	Min	33	34	34	32	31	29
		26%	27%	27%	26%	24%	23%
	Max	39	38	39	36	48	42
		31%	30%	31%	29%	38%	34%
Constraint Adjustors		11	10	10	11	8	10
		9%	8%	8%	9%	6%	8%
Total Bidders		126	128	125	123	127	124
Lower Bounded		5	7	7	8	6	8
		4%	5%	6%	7%	5%	6%
Experiment		30Min10%C	60Min10%C	5Min15%C	15Min15%C	30Min15%C	60Min15%C
Evaluators		5	5	9	5	4	4
		4%	4%	7%	4%	3%	3%
Bid Adjustors	Min	26	26	24	24	26	25
		21%	21%	20%	21%	20%	20%
	Max	8	9	10	12	11	12
		7%	7%	8%	10%	9%	10%
All Adjustors	Min	29	28	28	26	29	30
		24%	23%	23%	22%	23%	24%
	Max	43	42	41	39	46	43
		35%	35%	34%	33%	36%	35%
Constraint Adjustors		11	11	10	11	11	10
		9%	9%	8%	9%	9%	8%
Total Bidders		122	121	122	117	127	124
Lower Bounded		5	4	8	6	6	7
		4%	3%	7%	5%	5%	6%

Table C23 Winning Bidder Type Distribution - Activity Stopping Rule - Set 2

Winning Bidder Type Analysis Activity Stopping Rule (Set 2)							
Experiment		**5Min5%**	**15Min5%**	**30Min5%**	**60Min5%**	**5Min10%**	**15Min10%**
Evaluators		6	6	5	5	5	5
		5%	5%	4%	4%	4%	4%
Bid Adjustors	**Min**	24	22	25	22	21	22
		19%	18%	20%	18%	17%	18%
	Max	9	11	10	9	8	9
		7%	9%	8%	7%	6%	7%
All Adjustors	**Min**	32	38	35	35	31	30
		26%	31%	28%	28%	25%	24%
	Max	46	38	44	43	52	49
		37%	31%	35%	35%	41%	40%
Constraint Adjustors		8	7	8	9	9	9
		6%	6%	6%	7%	7%	7%
Total Bidders		125	122	127	123	126	124
Lower Bounded		5	6	7	4	3	4
		4%	5%	6%	3%	2%	3%
Experiment		**30Min10%**	**60Min10%**	**5Min15%**	**15Min15%**	**30Min15%**	**60Min15%**
Evaluators		5	6	10	6	5	6
		4%	5%	7%	5%	4%	5%
Bid Adjustors	**Min**	21	20	24	21	21	21
		17%	16%	17%	17%	16%	17%
	Max	9	11	13	12	11	12
		7%	9%	9%	10%	8%	10%
All Adjustors	**Min**	33	33	34	34	35	34
		26%	26%	24%	27%	27%	27%
	Max	48	48	54	43	49	43
		38%	38%	38%	35%	38%	35%
Constraint Adjustors		9	9	7	8	9	8
		7%	7%	5%	6%	7%	6%
Total Bidders		125	127	142	124	130	124
Lower Bounded		3	4	24	3	5	2
		2%	3%	17%	2%	4%	2%

Table C24 Winning Bidder Type Distribution - Minimum Revenue Stopping Rule

Winning Bidder Type Analysis							
Minimum Revenue Stopping Rule (Set 1)							
Experiment		**5Min5%A**	**15Min5%A**	**5Min10%A**	**15Min10%A**	**5Min15%A**	**15Min15%A**
Evaluators		5	5	5	5	4	5
		4%	4%	4%	4%	3%	4%
Bid Adjustors	**Min**	23	20	24	22	23	24
		18%	17%	20%	19%	19%	21%
	Max	14	13	12	13	15	13
		11%	11%	10%	11%	12%	11%
All Adjustors	**Min**	25	27	29	28	32	29
		20%	23%	25%	24%	26%	25%
	Max	47	43	38	39	41	38
		38%	36%	32%	33%	33%	32%
Constraint Adjustors		11	11	10	10	9	8
		9%	9%	8%	9%	7%	7%
Total Bidders		125	119	118	117	124	117
Lower Bounded		11	12	8	8	9	9
		9%	10%	7%	7%	7%	8%
Minimum Revenue Stopping Rule (Set 2)							
Experiment		**5Min5%A**	**15Min5%A**	**5Min10%A**	**15Min10%A**	**5Min15%A**	**15Min15%A**
Evaluators		5	6	7	7	5	6
		4%	5%	6%	6%	4%	5%
Bid Adjustors	**Min**	20	24	16	15	20	20
		17%	20%	15%	14%	16%	16%
	Max	8	10	13	13	15	15
		7%	8%	12%	12%	12%	12%
All Adjustors	**Min**	33	30	28	26	34	33
		27%	25%	26%	24%	27%	27%
	Max	46	44	35	39	44	42
		38%	36%	32%	36%	35%	34%
Constraint Adjustors		9	8	9	9	8	7
		7%	7%	8%	8%	6%	6%
Total Bidders		121	122	108	109	126	123
Lower Bounded		7	8	12	12	10	13
		6%	7%	11%	11%	8%	11%

Table C24 Winning Bidder Type Distribution - Maximum Round=10 Stopping Rule

Winning Bidder Type Analysis							
Maximum Round = 10 Stopping Rule (Set 1)							
Experiment		5M5%D10	15M5%D10	5M10%D10	15M10%D10	5M15%D10	15M15%D10
Evaluators		5	5	5	4	4	5
		4%	4%	4%	3%	3%	4%
Bid Adjustors	Min	27	26	28	25	25	24
		22%	20%	22%	20%	20%	19%
	Max	11	13	10	9	11	11
		9%	10%	8%	7%	9%	9%
All Adjustors	Min	33	34	27	29	29	27
		26%	27%	21%	24%	24%	22%
	Max	38	39	46	45	42	47
		30%	30%	37%	37%	34%	38%
Constraint Adjustors		11	11	10	11	11	11
		9%	9%	8%	9%	9%	9%
Total Bidders		125	128	126	123	122	125
Lower Bounded		7	8	4	4	6	10
		6%	6%	3%	3%	5%	8%
Maximum Round = 10 Stopping Rule (Set 2)							
Experiment		5M5%D10	15M5%D10	5M10%D10	15M10%D10	5M15%D10	15M15%D10
Evaluators		7	5	5	5	6	6
		5%	4%	4%	4%	4%	5%
Bid Adjustors	Min	24	24	21	22	22	21
		18%	20%	17%	18%	16%	16%
	Max	13	10	8	9	14	12
		10%	8%	6%	7%	10%	9%
All Adjustors	Min	35	36	31	29	39	32
		27%	30%	25%	24%	28%	25%
	Max	44	40	52	49	51	49
		33%	33%	41%	40%	36%	38%
Constraint Adjustors		9	7	9	9	8	9
		7%	6%	7%	7%	6%	7%
Total Bidders		132	122	126	123	140	129
Lower Bounded		14	9	3	5	6	7
		11%	7%	2%	4%	4%	5%

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I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy

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This dissertation was submitted to the Graduate Faculty of the Department of Decision and Information Sciences in the Warrington College of Business Administration and to the Graduate School and was accepted as partial fulfillment of the requirements for the degree of Doctor of Philosophy.

August 2000

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